$B = RA \Rightarrow b_i = Ra_i$

diff vector $d_i = Ra_i - b_i$

sum square of diffs

$$= \sum \|d_i\| = \sum d_i^T d_i$$

In matrix form

$$\begin{array}{c} n \times 2 \\ \begin{bmatrix} d_1^T \\ d_2^T \\ \vdots \\ d_n^T \end{bmatrix} \end{array} \begin{array}{c} 2 \times n \\ \begin{pmatrix} d_1 & d_2 & \cdots & d_n \end{pmatrix} \end{array} = \begin{array}{c} n \times n \\ \begin{bmatrix} d_1^T d_1 & d_1^T d_2 & \text{etc} & \\ d_2^T d_1 & d_2^T d_2 & & \\ & & \ddots & \\ \text{etc} & & & d_n^T d_n \end{bmatrix} \end{array}$$

we want sum of these
$\Rightarrow$ use Trace

$Tr(D^T D)$, where $D = RA - B$

$$E = Tr\, D^T D = Tr\,(RA-B)^T (RA-B)$$

$$= Tr\quad A^T \underbrace{R^T R}_{I} A - A^T R^T B - B^T RA + B^T B$$

$$E = Tr(A^T A + B^T B) - Tr(A^T R^T B) - Tr(B^T RA)$$

$$\frac{\partial E}{\partial R} = + R^T B A^T R^T - A B^T = 0$$

$$R^T B A^T R = A B^T$$

$$(R^T B A^T R^T)^T (R^T B A^T R^T) = (AB^T)^T (AB^T)$$

$$R A B^T \cancel{R R^T} B A^T R^T = (AB^T)^T (AB^T)$$

$S = AB^T$ $\quad R S S^T R^T = S^T S$

Both sides symmetric

Both sides have same eigenvalues!

Now use eigendecomposition...

$S = AB^T \qquad \underbrace{RSS^TR^T}_{WDW^T} = \underbrace{S^TS}_{VDV^T}$

D diagonal (eigenvalues)
W orthogonal ($W^TW = WW^T = I$)
V orthogonal ($V^TV = VV^T = I$)

$$RWDW^TR^T = VDV^T$$

$$(RW)D(RW)^T = VDV^T$$

$$\Rightarrow \quad RW = V$$

$$R = VW^T$$