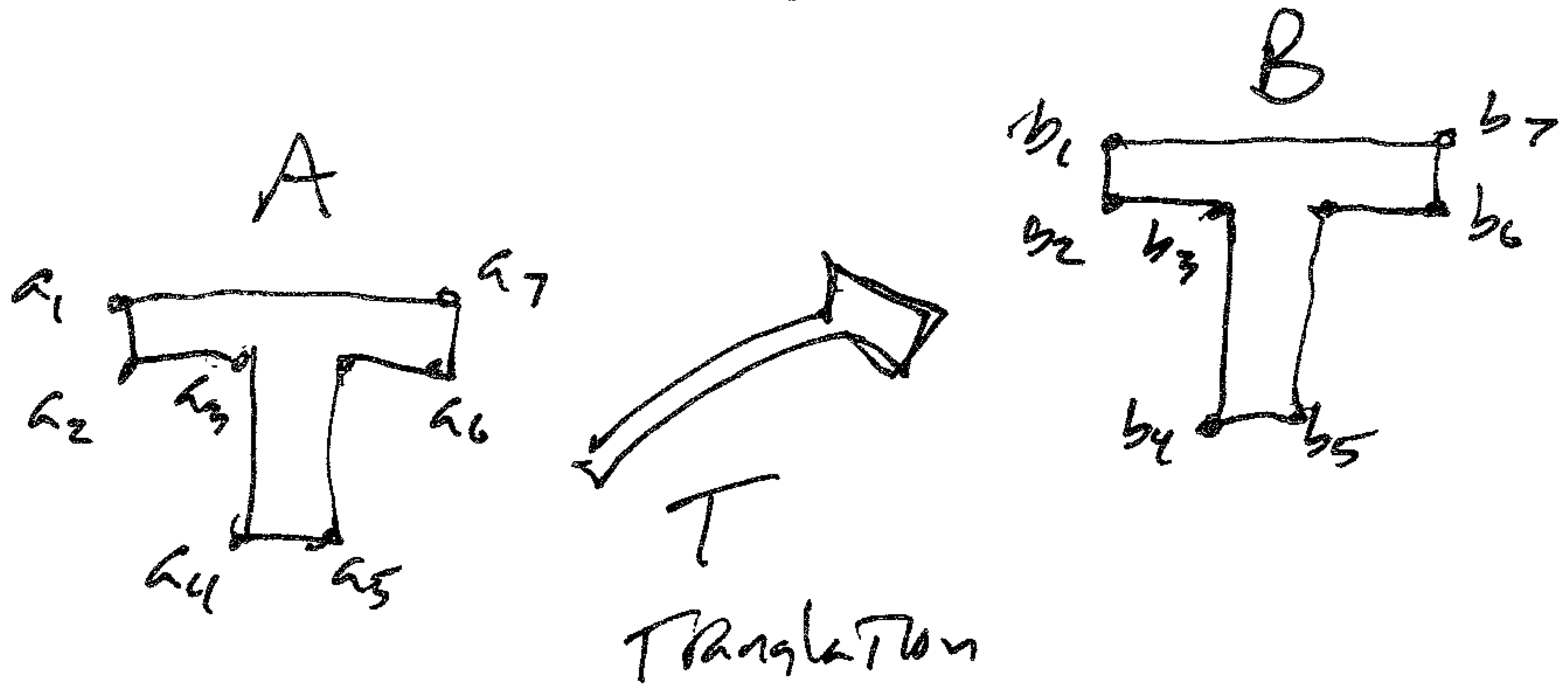


$$A = [a_1 \; a_2 \cdots a_n] \qquad B = [b_1 \; b_2 \cdots b_n]$$

Least squares

$$B = T(A) = A + T$$

$$\Rightarrow b_i = a_i + T$$

$$E = \sum_{i=1}^{n} (a_i + T - b_i)^T (a_i + T - b_i)$$

$$= \sum (a_i - b_i)^T (a_i - b_i) + 2T^T (a_i - b_i) + T^T T$$

$$\frac{\partial E}{\partial T} = \cancel{2} \sum (a_i - b_i) + \cancel{2} \sum T = 0$$

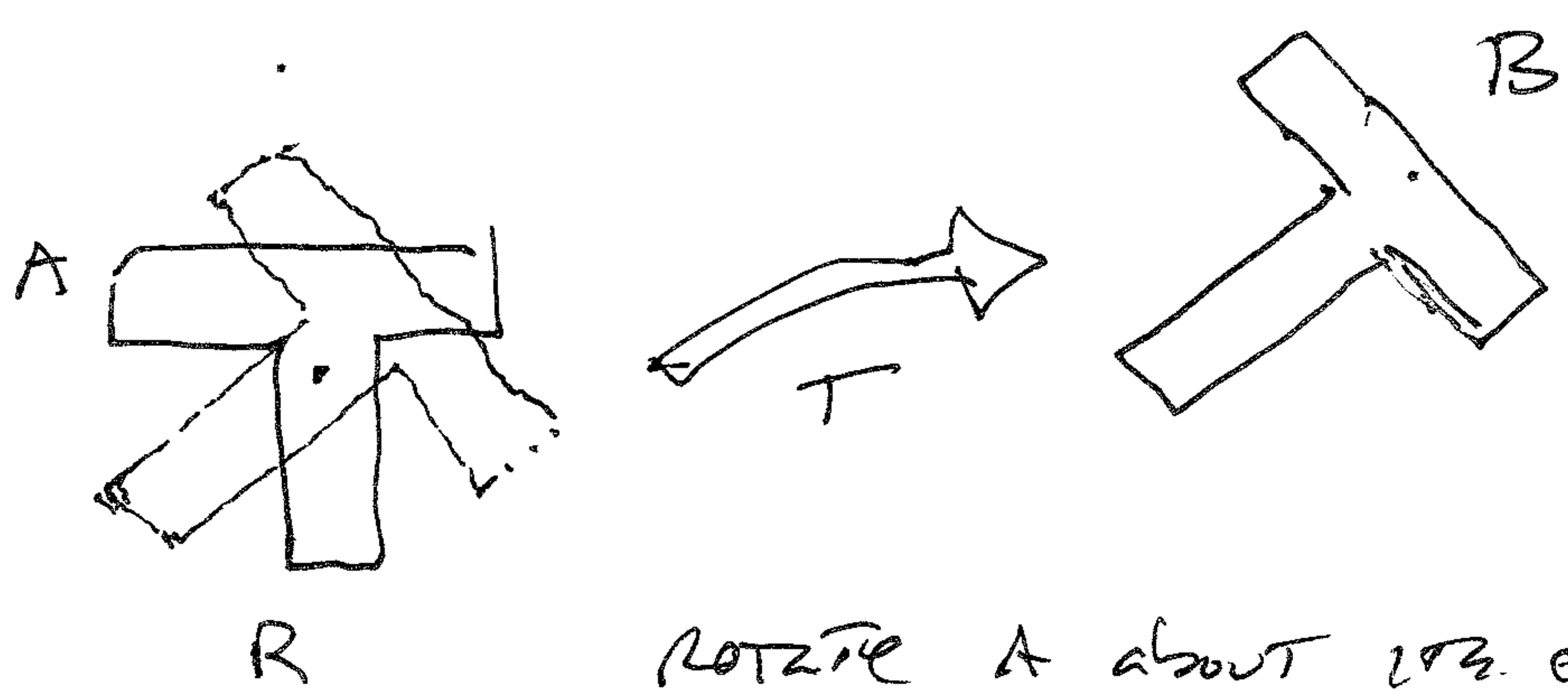
$$\sum T = \sum (b_i - a_i)$$

$$n T = \sum b_i - \sum a_i$$

$$T = \sum b_i / n - \sum a_i / n$$

$$T = \bar{B} - \bar{A}$$

Align centers of mass



ROTATE A about its center of mass and then translate, to get B

$$B = T(A) = R(A - \bar{A}) + \bar{A} + T$$

$$b_i = R(a_i - \bar{A}) + \bar{A} + T$$

$$E = \sum \left[R(a_i - \bar{A}) + \bar{A} + T - b_i\right]^T \left[R(a_i - \bar{A}) + \bar{A} + T - b_i\right]$$

$$\frac{\partial E}{\partial T} = 2\sum \left[R(a_i - \bar{A}) + \bar{A} + T - b_i\right] = 0$$

$$= \sum R a_i - nR\bar{A} + n\bar{A} + nT - \sum b_i = 0$$

$$T = \frac{\sum b_i}{n} - \bar{A} + \cancel{R\bar{A}} - \underbrace{R\frac{\sum a_i}{n}}_{\cancel{R\bar{A}}}$$

$$T = \bar{B} - \bar{A}$$

so still centers of mass align, regardless of the rotation...

So now, assume both shapes are centered at the origin, so $\bar{A} = \bar{B} = 0$, and compute just rotation between them...

$$B = RA \Rightarrow b_i = R a_i$$