Consistency with External Knowledge: The TopDown Algorithm

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Simons Privacy Workshop

(revised slides)

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Disclaimer

All opinions, statements, conclusions, etc., in this talk are my own (as a researcher on differential privacy), and are not the official position of the U.S. Census Bureau.

Outline

Goal

- DAS: disclosure avoidance system
- Publish a histogram with billions of cells using formal privacy.
  - Location (hierarchical) - National, State, County, Tract, Block Group, Block. $\approx$ 6 million blocks
  - Ethnicity: 2 values
  - Race: 63 values
  - Voting age: 2 values
  - Residence type ("household" or group quarters code) - 8 values
- Hierarchical workload
  - Counting queries about demographics in each geographic region
  - E.g., 2010 PL94-171 Redistricting and Advanced Group Quarters Summary Files
- The data are sparse
  - $\approx$ 12 billion cells
  - $\approx$ 309 million people
  - Workload: 641 non-identity queries per geo-unit $\approx$ 3.6 billion queries
  - $+$12 billion identity queries

Formal Privacy

- Differential Privacy

Definition (Differential Privacy (DMNS06))

Let $\epsilon > 0$. An algorithm M satisfies ϵ-differential privacy if for all $\omega \in \text{range}(M)$ and all pairs of databases D_1, D_2 that differ on the value of one page of Census questionnaire (information about 1 person),

$$P(M(D_1) = \omega) \leq e^{\epsilon} P(M(D_2) = \omega)$$

- Note: multiple tables
- Person demographics: 1 person affects 1 row.
- Households/Housing units: 1 person can modify 1 row in a bounded way (different from Uber's model)
- Group Quarters: similar to households
- Geographic boundaries: no protection

Requirements

- Create microdata
  - Ensures that published "universe person" tabulations are mutually consistent.
  - Also system requirement: output of DAS goes into tabulation system.
  - Equivalent to histogram with nonnegative integer entries.
- Run within X days
  - Implemented in Spark
  - Uses GovCloud
  - Use commercial-grade optimizers (e.g., Gurobi, CPLEX)
- Run before all data are available
  1. PL94-171 first
  2. Summary File 1
  3. Urban/Rural update
  4. etc.
- Consistent with external pieces of knowledge
- Consistent with prior releases

Consistency with External Knowledge

- Some datasets are treated as effectively public.
  - Local Update of Census Addresses Operation (LUCA) dataset contains # of housing units and GQ units of each type in each block.
  - Number of occupied GQ facilities of each type in each block assumed to be known.
- Some information might be declared public as policy decision.
  - In 2010: population of each block.
  - In 2010: number of occupied housing units in each block
  - # occupied housing units = # of householders
- Invariants:
  - Queries in true data that must have same answers in "privatized" data.
  - Differentially private algorithms are still differentially private.
  - Privacy semantics, however, are awkward.
  - Easily make simple problems NP hard.
- Structural zeros:
  - Data-independent restrictions
  - 0 householders aged 14 and under
  - # householders $\geq$ # spouses + # unmarried partners of householders.

Invariants and Utility

- Invariants may be forced by policy decisions.
- Invariants based on external knowledge can increase trust in the microdata.
- Utility:
  - Making published data consistent with the invariants could increase accuracy of microdata.
  - In experiments, feasible datasets (satisfying invariants) can be very different from unrestricted datasets (given the same noisy measurements).

The Spherical Cows

- Incremental Schema Extension - Incrementally add columns to DP microdata
- e.g., start with Race (R), Ethnicity (E), Voting Age status (VA)

R	E	VA

→

R	E	VA	State

→

R	E	VA	State	County

→ etc.

  - Necessary because not all data are available at once.
  - Also useful for scalability.
    - Microdata generation: measure then postprocess
    - Cannot fit postprocessing optimization problem in memory
- Consistency with External Knowledge
  - Linear constraints on histogram constructed from full schema.
  - Ensure there exists an extension of | R | E | VA | that will satisfy those constraints.
  - Decision problem (microdata are consistent?) is NP complete.

Outline

TopDown Framework (without invariants)

- Histogram is too big to fit in memory, must be created in pieces.
- First generate nonnegative integer histogram H at the national level.
- Create child histograms H_i for each state S_i, with $\sum_i H_i = H$.
- Recursively create county, tract, block group, block level histograms.
- Number of optimization problems increases down the hierarchy
- Size of optimization problems decreases
  - Algorithm estimates which counts are nonzero
  - Splits these counts among children
  - Variables that are 0 at the parent are dropped from future optimizations.

National Level Histogram H

- Total U.S. population is not protected.
- Given linear query workload W, use High-dimensional matrix mechanism to obtain [MMHM2018] linear queries Q to ask.
- Obtain noisy measurements $M = Q(H)$+Noise
- Solve $H^* = \arg\min_{H^*} ||Q(H^*) - M||_2^2$ s.t. $sum(H^*) = n$ and $H^* \succeq 0$
  - Now we have a nonegative fractional histogram of population demographics.

National Histogram Linear solve

- Nonnegative fractional histogram H^*.
- Round using LP

$$\begin{aligned}
\arg\min_{\widetilde{H}} & ||\widetilde{H} - H^*||_1 \\
\text{s.t. } & \widetilde{H} \succeq 0 \text{ (nonnegativity)} \\
& |\widetilde{H}[x] - H^*[x]| \leq 1 \text{ for all cells } x \\
& \sum_x \widetilde{H}[x] = \sum_x H^*[x] \text{ (total sum constraint)}
\end{aligned}$$

- Constraint matrix is Totally Unimodular (TUM).
- Many LP algorithms (barrier+crossover, simplex) give integer solutions.
- To be safe, implementation asks Gurobi to solve IP instead of LP (fast because of TUM)

State Level Histograms

- Now we have a nonnegative integer histogram $\widetilde{H}$
  - National level demographics
  - Equivalent to microdata with no geography
- Next we add States + DC.
  - H_i: demographics histogram for state i
    - Ignore cells that are 0 at national level DP histogram $\widetilde{H}$
    - Reduces size of the optimization problem.
  - Given workload at each state + DC, use HDMM to obtain linear queries Q to ask.
  - Noisy measurement for state i: $M_i = Q(H_i)+\text{Noise}$
  - Then we solve an L_2 followed by L_1 optimization problem.

State Level Histograms: L_2 solve

- $\widetilde{H}$ is national level DP histogram
- Noisy state level measurements $M_1, \ldots, M_{51}$
- Obtain DP state-level nonnegative fractional histograms that add up to $\widetilde{H}$

$$\arg\min_{H_1^*,\ldots,H_m^*} \sum_{j=1}^{m} ||Q(H_j^*) - M_j||_2^2$$

$$\text{s.t. } H_j^* \succeq 0 \quad \text{for all } j$$

$$\sum_{j=1}^{m} H_j^* = \widetilde{H}$$

State Level Histograms: Linear solve

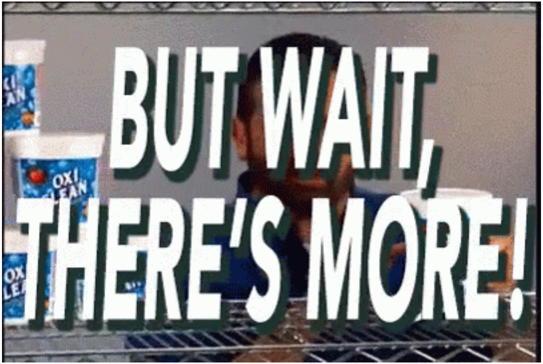
- Now round using IP that is equivalent to LP when using e.g., barrier+crossover or simplex algorithms.
- H_j^* are nonnegative fractional state level histograms

$$\begin{aligned} \arg \min_{\widetilde{H}_1,\dots,\widetilde{H}_m} \sum_{j=1}^{m} ||\widetilde{H}_j - H_j^*||_1 \\ \text{s.t. } \widetilde{H}_j \succeq 0 \text{ for all } j \\ |\widetilde{H}_j[x] - H_j^*[x]| \leq 1 \text{ for all } j \text{ and cells } x \\ \sum_j \widetilde{H}_j = \widetilde{H} \end{aligned}$$

Then Recurse

- (In parallel) For each state, we generate its county level histograms.
- For each county, generate its tract histograms.
- For each tract, generate its block level histograms.
- Convert back to microdata.
- $\approx 20k$ lines of code
- $\approx 60k$ more lines of supporting code

TopDown Algorithm

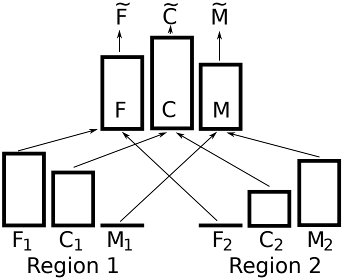


Outline

Invariants

- Final data (with all fields) must satisfy (mostly) linear constraints.
- Consumed most time & effort.
  - Semantics:
    - What is impact on privacy if some exact statistics about data are published?
    - How do privacy semantics change?
    - Needed for policy decisions.
    - Short answer: it's complicated.
  - Algorithm:
    - How do we enforce them in DP microdata?
    - Short answer: it's complicated.

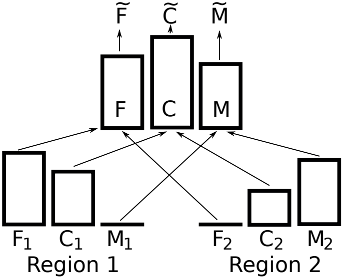
An Example (1)



- Small college town, 2 regions
- Every student lives in dorms
  - Male-only (M)
  - Female-only (F)
  - Co-ed (C)
- Knowledge:
  - 100 students in each region: $F_1 + C_1 + M_1 = F_2 + C_2 + M_2 = 100$
  - All dorms are occupied.
  - R_1 : 0 Male, 1 Female, 1 Co-ed dorms: $M_1 = 0$; $F_1 \geq 1$; $C_1 \geq 1$.
  - R_2 : 1 Male, 0 Female, 1 Co-ed dorms: $M_2 \geq 1$; $F_2 = 0$; $C_2 \geq 1$
- We already generated town-wide DP statistics: $\widetilde{F}, \widetilde{C}, \widetilde{M}$.
- Consistent with background knowledge?

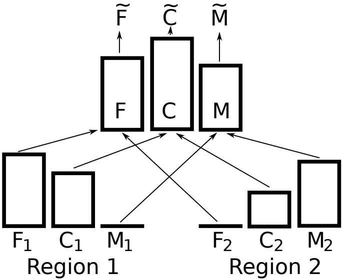
An Example (2)

- Knowledge:
  - 100 students in each region: $F_1 + C_1 + M_1 = F_2 + C_2 + M_2 = 100$
  - All dorms are occupied.
  - R_1 : 0 Male, 1 Female, 1 Co-ed dorms: $M_1 = 0; F_1 \geq 1; C_1 \geq 1.$
  - R_2 : 1 Male, 0 Female, 1 Co-ed dorms: $M_2 \geq 1; F_2 = 0; C_2 \geq 1$
- Consistency: implications for $\widetilde{F}, \widetilde{C}, \widetilde{M}$?



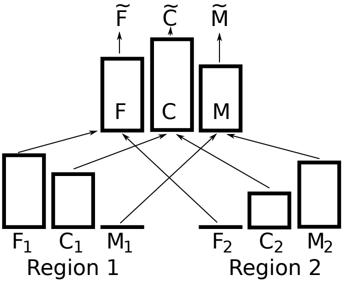
An Example (3)

- Knowledge:
  - 100 students in each region: $F_1 + C_1 + M_1 = F_2 + C_2 + M_2 = 100$
  - All dorms are occupied.
  - R_1 : 0 Male, 1 Female, 1 Co-ed dorms: $M_1 = 0; F_1 \geq 1; C_1 \geq 1.$
  - R_2 : 1 Male, 0 Female, 1 Co-ed dorms: $M_2 \geq 1; F_2 = 0; C_2 \geq 1$
- Consistency: implications for $\widetilde{F}, \widetilde{C}, \widetilde{M}$?
  - $\widetilde{M} \geq 1$
  - $\widetilde{F} \geq 1$
  - $\widetilde{C} \geq 2$
  - $\widetilde{F} + \widetilde{C} + \widetilde{M} = 200$
  - Are we done?

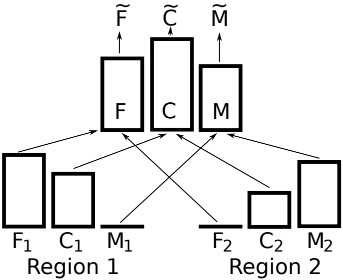


An Example (4)

- Knowledge:
  - 100 students in each region: $F_1 + C_1 + M_1 = F_2 + C_2 + M_2 = 100$
  - All dorms are occupied.
  - R_1 : 0 Male, 1 Female, 1 Co-ed dorms: $M_1 = 0; F_1 \geq 1; C_1 \geq 1$.
  - R_2 : 1 Male, 0 Female, 1 Co-ed dorms: $M_2 \geq 1; F_2 = 0; C_2 \geq 1$
- Consistency: implications for $\widetilde{F}, \widetilde{C}, \widetilde{M}$?
  - $\widetilde{M} \geq 1,\ \widetilde{F} \geq 1,\ \widetilde{C} \geq 2,$ $\widetilde{F} + \widetilde{C} + \widetilde{M} = 200$, ??
- Suppose $\widetilde{F} = 49,\ \widetilde{C} = 50,\ \widetilde{M} = 101$
  - Satisfies these constraints
  - But, only 1 male-only dorm.
  - It is in region with 100 students.
  - $\therefore\ \widetilde{M} = 101$ is not valid



An Example (5)



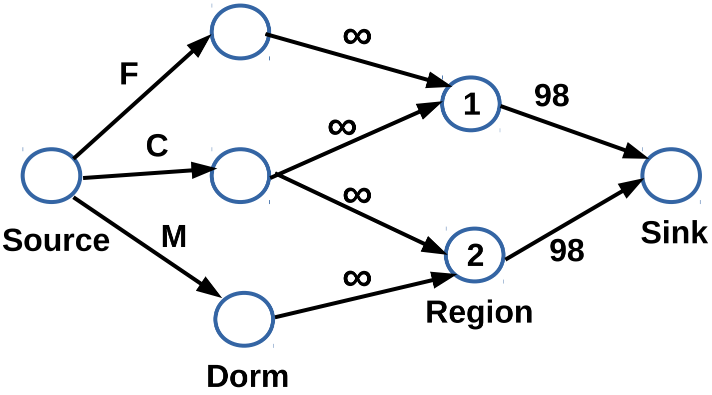
- Knowledge:
  - 100 students in each region: $F_1 + C_1 + M_1 = F_2 + C_2 + M_2 = 100$
  - All dorms are occupied.
  - R_1 : 0 Male, 1 Female, 1 Co-ed dorms: $M_1 = 0; F_1 \geq 1; C_1 \geq 1$.
  - R_2 : 1 Male, 0 Female, 1 Co-ed dorms: $M_2 \geq 1; F_2 = 0; C_2 \geq 1$
- Consistency: implications for $\widetilde{F}, \widetilde{C}, \widetilde{M}$?
- The necessary and sufficient constraints (auto-proved via FME):

$$\widetilde{F} \geq 1 \qquad \widetilde{C} \geq 2 \qquad \widetilde{M} \geq 1$$

$$\widetilde{F} \leq 99 \qquad \widetilde{C} + \widetilde{F} \geq 101 \qquad \widetilde{C} + \widetilde{F} + \widetilde{M} = 200$$

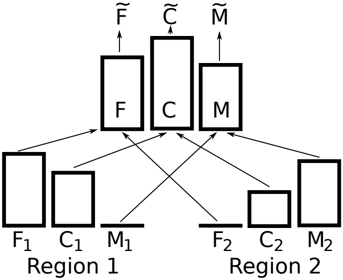
via Network Flows

- Reduction to Network Flow (change $\geq c$ constraints to ≥ 0)
- Use max-flow/min-cut theorem



Sphering The Cow

- Starting schema: S_0 (set of table columns)
  - e.g, { Dorm Type }
- Extended schema $S \supset S_0$
  - e.g., {Dorm Type, Region}
- T_0: microdata table with schema S_0
- T: microdata table with schema S
- C: set of constraints on T
  - Total population in each region
  - Presence/absence of occupied dorms
- C_0: set of constraints on T_0
  - What we want
  - Constraints on population in each dorm in T_0



Implied constraints

Definition (Necessary Constraints)

C_0 is necessary if $C(T)$ =true $\Rightarrow C_0(T_0)$ =true, where T_0 is projection of T onto the attributes in schema S_0

Definition (Sufficient Constraints)

C_0 is sufficient if $C_0(T_0)$ =true $\Rightarrow$ there exists an extension T of T_0 with $C(T)$ =true

We want C_0 to be necessary and sufficient:

- $\widetilde{T}_0$: DP microdata
- Sufficient: If $C_0(\widetilde{T}_0) =$ true, we can always add columns to get a DP version $\widetilde{T}$ that satisfies C
- Necessary: Constraints are not too restrictive (do not add unnecessary bias)

Implied Constraints

- How do we find them?
- NP-complete in universe size when $|S_0| = 2$ and $|S| = 3$. Easily encodes 3-SAT
- NP-complete if each region only has equality constraints for 2 one-way marginals
  - NP-complete in # of regions and size of one of the marginals (if 2nd marginal has size 3)

Region A

	$\mathbf{R_V = 0}$	$\mathbf{R_V = 1}$	
$\mathbf{R_H = 0}$	?	?	6
$\mathbf{R_H = 1}$	?	?	16
	17	5	

Region B

	$\mathbf{R_V = 0}$	$\mathbf{R_V = 1}$	
	?	?	15
	?	?	5
	5	15	

- But exists an inefficient algorithm if constraints are linear:
  - Fourier-Motzkin elimination (FME).
  - Double-exponential complexity (Can be accelerated but not for our scale)
  - Works for fractional histograms (often provable for integer histograms).

Outline

State Level Histograms: L_2 solve with invariants

- $\widetilde{H}$ is national level DP histogram
- Compute implied constraints C_i for each state i
- Noisy state level measurements $M_1, \ldots, M_{51}$
- Obtain DP state-level nonnegative fractional histograms that add up to $\widetilde{H}$

$$\arg\min_{H_1^*,\ldots,H_m^*} \sum_{j=1}^{m} ||Q(H_j^*) - M_j||_2^2$$

$$\text{s.t. } H_j^* \succeq 0 \quad \text{for all } j$$

$$C_i(H_j^*) = \text{true} \quad \text{for all } j$$

$$\sum_{j=1}^{m} H_j^* = \widetilde{H}$$

State Level Histograms: Linear solve with invariants

- This rounding using IP that is equivalent to LP when using barrier+crossover or simplex algorithms.
  - Under conditions like TUM constraint matrix or nice obj + rhs
- H_j^* are nonnegative fractional state level histograms

$$\arg\min_{\widetilde{H}_1,\dots,\widetilde{H}_m} \sum_{j=1}^{m} ||\widetilde{H}_j - H_j^*||_1$$

$$\text{s.t. } \widetilde{H}_j \succeq 0 \text{ for all } j$$

$$|\widetilde{H}_j[x] - H_j^*[x]| \leq 1 \text{ for all } j \text{ and cells } x$$

$$C_i(\widetilde{H}_j) = \text{true} \quad \text{for all } j$$

$$\sum_j \widetilde{H}_j = \widetilde{H}$$

TopDown with Invariants

- Implied constraints deduced by hand + FME
- L_2 solve: creates nonnegative fractional histogram
  - Implied constraints C_0 are added to the problem.
  - Implies fractional feasible extension exists.
- L_1 solve: rounds to nonnegative integer counts.
  - Generally, linear implied constraints do not always guarantee feasible integer solution
  - They do if the problem constraint matrix is TUM (then linear solve is also usually fast)
  - Some of our implied invariant constraints are not TUM
    - But integer optimal solution exists
    - Solve is slow
    - Possibly equivalent to TUM constraints (network flow and a few others)

Example

- 3 digit GQ code of occupied group quarters might be invariant
  - Similar to college dorm example
  - But 28 types of GQ
  - In general, $\approx 2^{28}$ implied constraints, one for each combination of GQ.
  - Can be much smaller, depending on data.
  - For each combination S of GQ:
    - Total population living in GQ of types in S is $\leq c$
    - c depends on total population in blocks that have GQ types from S
  - Constraint matrix is not TUM
    - Might be equivalent to TUM (via network flows)
    - Network flow integrality theorem says an integer solution exists

Workarounds

- "The Failsafe"
  - In the worst case, breaks out of the framework.
  - If a solve fails (or is slow) in, e.g., county level histogram H_c
    - Cannot find feasible tract histograms $H_1, \dots, H_k$ with $\sum_i H_i[x] = H_c[x]$ for all x
    - Drop this requirement
    - Use weaker requirements (e.g., total population matches: $\sum_i \sum_x H_i[x] = \sum_x H_c[x]$) and other tricks
    - Generate tracts
    - The county is changed to the sum of the tracts
    - Worse accuracy but invariants maintained
- "Minimal Schema"
  - S_0: smallest set of attributes that cover the invariants + all geography.
  - Generate nonnegative integer histogram in 2 solves L_2 followed by L_1.
    - Simultaneously for all levels of geography, estimate group quarters population by GQ type (nothing else)
  - Then extend to the other attributes.
  - Works if these problems fit in memory
- Cutting plane: find the instance-level necessary constraints

Current Invariants

- Have explored many invariants.
- Choice of invariants is policy decision.
  - Policy can be affected by privacy semantics
  - Policy can be affected by computational difficulty
- Current set of invariants being explored:
  - State population totals are invariant.
  - # occupied GQ facilities of each type in each block are invariant.
  - Total # of housing units in each block are invariant.
  - Auxiliary information about GQ (age restrictions, female-only, male-only, co-ed).
  - Also structural zeros.
- Historical invariants deducible from `https://www.census.gov/content/dam/Census/library/working-papers/2018/adrm/Disclosure%20Avoidance%20for%20the%201970-2010%20Censuses.pdf`

Outline

RDP/zCDP

- Currently using pure DP with Laplace noise and geometric mechanism
- Planning experiments with Gaussian noise and RDP/zCDP.
- Choice of Gaussian variance via reductions from RDP/zCDP to (ϵ, δ)-differential privacy.
- How to choose failure probability?
- Conservative: $\delta = 10^{-14}/4$
  - $\approx 4 * 10^8$ people
  - $\approx 10^{-6}$ chance of failure
  - Based on (ϵ, δ)-DP algorithm that returns a random record with probability 10^{-6}
- Moderate: $\delta = 10^{-6}$
  - Rough interpretation: each bit of a person's record has probability 10^{-6} of getting less privacy than ϵ-differential privacy

RDP/zCDP

- For $\delta = 10^{-14}$ (conservative value)
- Moment accountant privacy budget split across 6 levels of geographic hierarchy.
- For identity queries, noise variance

ϵ	Laplace Variance	Gaussian Variance
1	288.0	785.6
2	72.0	199.4
3	32.0	89.9
4	18.0	51.3
5	11.5	33.3

RDP/zCDP

- For $\delta = 10^{-9}$ (intermediate conservative value)
- Moment accountant privacy budget split across 6 levels of geographic hierarchy.
- For identity queries, noise variance:

ϵ	Laplace Variance	Gaussian Variance
1	288.0	509.3
2	72.0	130.3
3	32.0	59.2
4	18.0	34.0
5	11.5	22.2

RDP/zCDP

- For $\delta = 10^{-6}$ (moderate value)
- Moment accountant privacy budget split across 6 levels of geographic hierarchy.
- For identity queries, noise variance:

ϵ	Laplace Variance	Gaussian Variance
1	288.0	343.5
2	72.0	88.8
3	32.0	40.7
4	18.0	23.6
5	11.5	15.6

RDP/zCDP

- Gaussian variance is larger than Laplace
- But tails are lighter (fewer outliers)
- May affect postprocessing steps
- Might have better tuned query workload
- So experiments are planned (but many other problems need solving)
- Most likely scenario:
  - Use pure differential privacy
  - Report corresponding RDP/zCDP parameters using reductions from ϵ-differential privacy to RDP/zCDP

Thank You