Homework 9 – Due Thursday, November 10, 2016 on Canvas

Please refer to HW guidelines from HW1, course syllabus, and collaboration policy.

General guidelines for reductions Model your solutions on the reduction of Maximum Matching to Maximum Flow given in class. To reduce problem B to problem A:

1. Explain how to transform an instance $\mathcal{I}_B$ of B into an instance $\mathcal{I}_A$ of A.
2. Explain how to transform a solution $\mathcal{S}_A$ for $\mathcal{I}_A$ into a solution $\mathcal{S}_B$ for $\mathcal{I}_B$.
3. (**large fraction of the points**) Prove that $\mathcal{S}_B$ is a correct solution for $\mathcal{I}_B$, provided that $\mathcal{S}_A$ is a correct solution for $\mathcal{I}_A$. In case of optimization problems, it usually involves proving that the value of $\mathcal{S}_A$ is equal to the value of $\mathcal{S}_B$. (Often, it is easier to prove $\geq$ and $\leq$ separately.)
4. Analyze the efficiency of the resulting algorithm for problem B that uses your reduction and the most suitable algorithm for problem A that we studied. Make sure that the running time is expressed in terms of the length of $\mathcal{I}_B$, not $\mathcal{I}_A$.

Exercises These should not be handed in, but the material they cover may appear on exams:

1. Reduce the maximum flow problem for a network with several source nodes $(s_1, \ldots, s_k)$ and several sink nodes $(t_1, \ldots, t_\ell)$ into the single-source single-sink maximum flow problem.
2. Some networks have capacity constraints on the flow amounts that can flow through their intermediate vertices. Explain how the maximum flow problem for such a network can be reduced to Maximum Flow with edge capacity constraints only.
3. (**Perfect matching in a k-regular graph**) You are given a bipartite graph G with vertex sets L and R and $k \geq 1$.
   (a) Prove that if every node in L and R has degree exactly k then G has a perfect matching. Deduce (i.e., prove assuming the previous statement) that such a graph has k disjoint perfect matchings.
   (b) Show that the first statement you are asked to prove in part (a) is false when "degree exactly k" is replaced with "degree at least k".
4. As usual, read, solve and check your answers on the solved exercises in Chapter 7. They might be helpful for some of the homework problems.

Problems to be handed in

1. (**Ergonomic design**) Chapter 7, problem 6.
2. (k **edge-disjoint paths**) Chapter 7, problem 32.
3. (**Cycle cover**) A *cycle cover* of a given directed graph $G = (V, E)$ is a set of vertex-disjoint cycles that cover all the vertices. Describe and analyze an efficient algorithm to find a cycle cover for a given graph, or correctly report that no cycle cover exists. Base your algorithm on a reduction to bipartite matching.