Intro to Theory of Computation

CS 464

LECTURE 27

Last time

- Polynomial-time reductions
- NP-completeness
- Cook-Levin theorem

Today

- Examples of NP-complete problems

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The class NP

NP is the class of languages that have polynomial-time verifiers.

I-clicker question (frequency: AC)

Recall: The running time of a verifier V($\langle w, c \rangle$) is measured only in terms of length of w.

If we allowed a verifier to run in time polynomial in the length of $\langle w, c \rangle$, the class NP would

A. **be smaller**

B. **be the same**

C. **contain more (decidable only) languages**

D. **contain more (even undecidable) languages**

E. **none of the above**

I-clicker question (frequency: AC)

3SAT is an example of a problem that cannot be solved by an algorithm.

- **A.** True
- **B.** False
- **C.** It is an open question
- **D.** None of the above

I-clicker question (frequency: AC)

3SAT is an example of a problem that cannot be solved by a polynomial time algorithm.

A. True

B. False

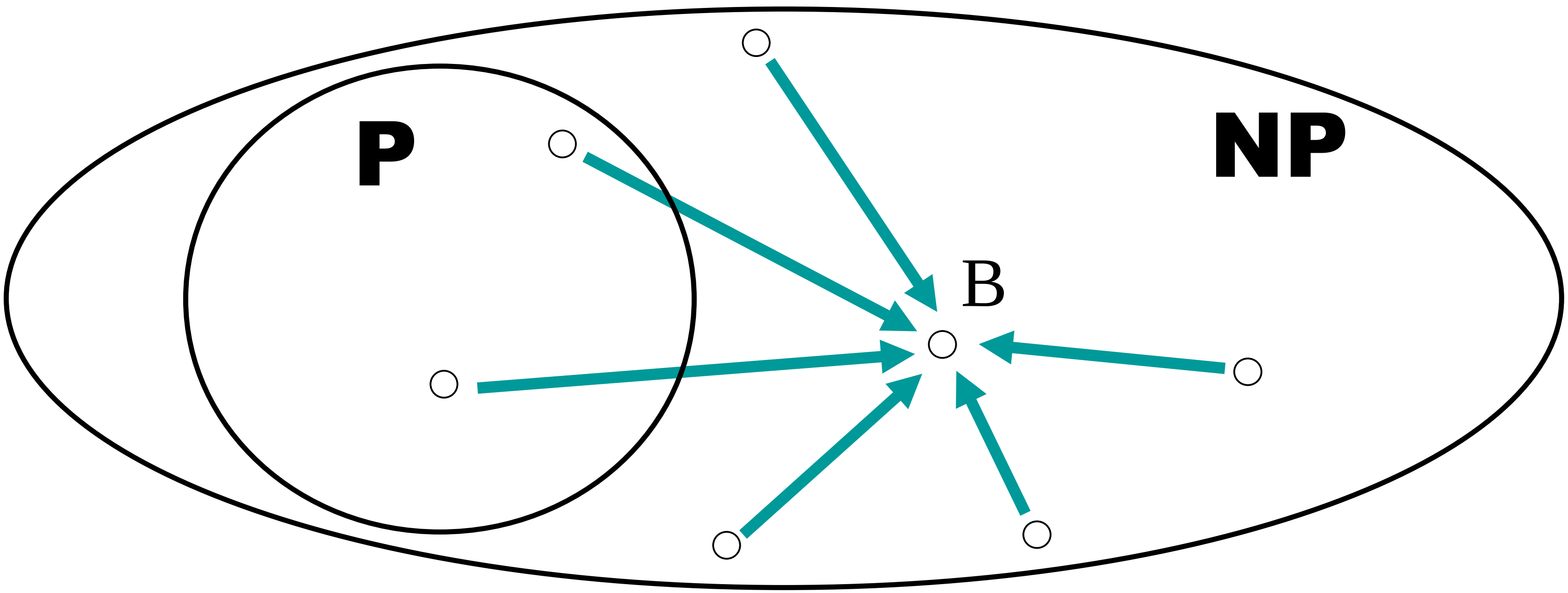
C. It is an open question

D. None of the above

Hardest problems in NP

A language B is **NP-complete** if

1. B∈ **NP**
2. B is **NP-hard**,i.e.,
   every language in NP is poly-time reducible to B.



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Establishing NP-completeness

- Once we establish first "natural" NP-complete problems, others fall like dominoes.

- Recipe to establish NP-completeness of problem Y.
  - Step 1. Show that Y is in NP.
  - Step 2. Choose an NP-complete problem X (e.g., 3SAT).
  - Step 3. Prove that $X \leq_p Y$.

SUBSET-SUM

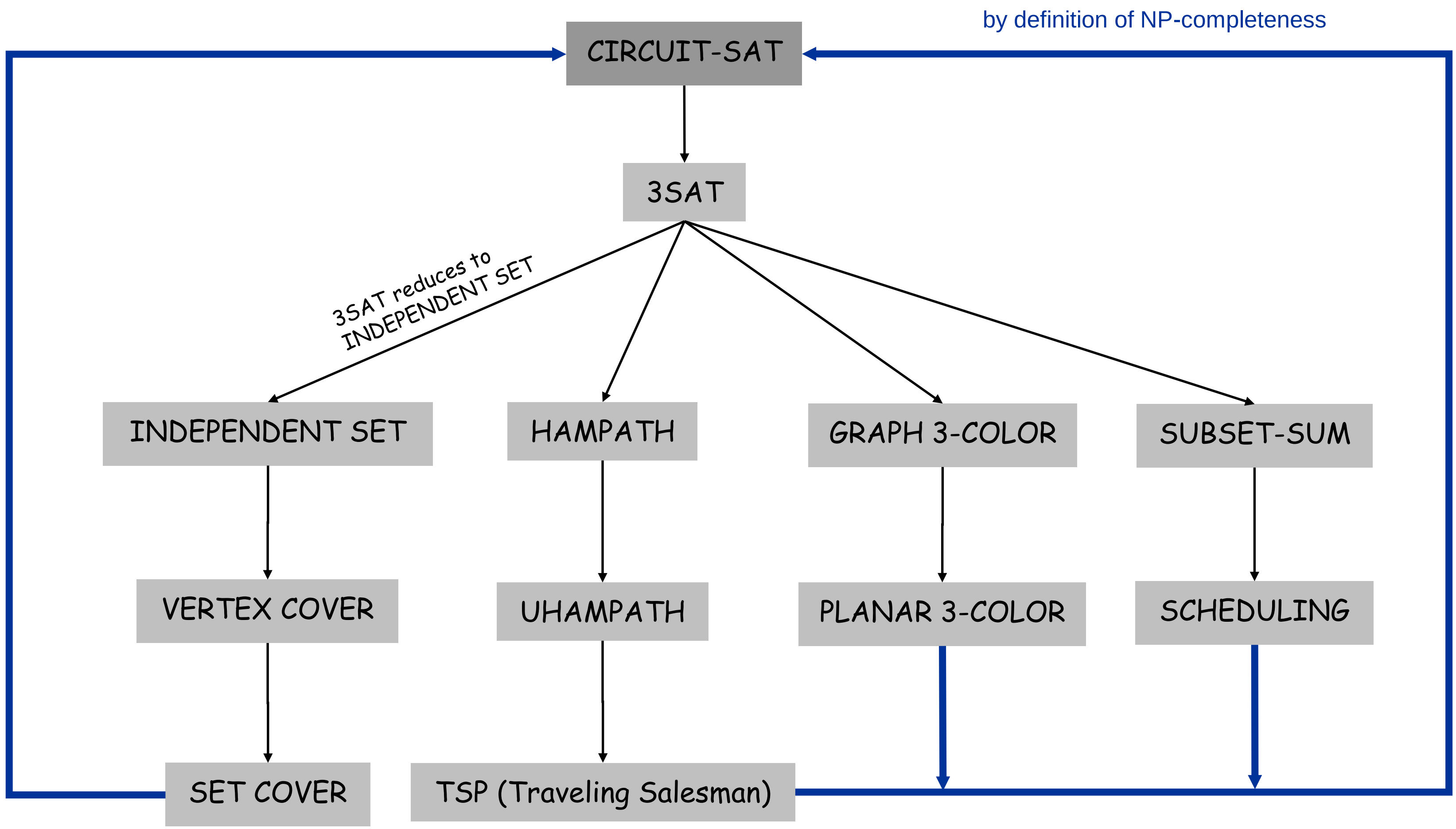
- SSUM = $\{\langle S, t\rangle \mid S = \{x_1, \ldots, x_r\},$ and for some $\{y_1, \ldots, y_{r'}\} \subseteq \{x_1, \ldots, x_r\},$ we have $y_1 + \cdots + y_{r'} = t\}$
- Repetitions allowed.
- Examples: $\langle\{5, 7, 23\}, 46\}\rangle \in SSUM$
  $\langle\{5, 15, 25\}, 46\}\rangle \notin SSUM$
- SSUM ∈ NP Certificate: $y_1, \ldots, y_{r'}$

3SAT$\leq_p$SUBSET-SUM

- SSUM = $\{\langle S, t\rangle \mid S = \{x_1, \dots, x_r\}$, and for some $\{y_1, \dots, y_{r'}\} \subseteq \{x_1, \dots, x_r\}$, we have $y_1 + \cdots + y_{r'} = t\}$
- ``On input $\langle \phi \rangle$, where ϕ is a 3CFN formula with variables $x_1, \dots, x_\ell$ and clauses $c_1, \dots, c_k$,
- Output a set S of numbers and target number t
- (constructed on the board) ''

NP-Completeness

All problems below are NP-complete and hence poly-time reduce to one another!



Some NP-Complete Problems

- Six basic genres of NP-complete problems and paradigmatic examples.
  - Packing problems: SET-PACKING, INDEPENDENT SET.
  - Covering problems: SET-COVER, VERTEX-COVER.
  - Constraint satisfaction problems: SAT, 3-SAT.
  - Sequencing problems: HAMPATH, TSP.
  - Partitioning problems: 3D-MATCHIN,G 3-COLOR.
  - Numerical problems: SUBSET-SUM, KNAPSACK.
- Most NP problems are either known to be in P or NP-complete.
- Notable exceptions. Factoring, graph isomorphism, Nash equilibrium.

An encyclopedia of NP-complete problems