Intro to Theory of Computation

LECTURE 23

CS 464

Last time

- Recursion theorem
- Measuring complexity
- Asymptotic notation
- Relationship between models

Today

- Relationship between models: deterministic/nondeterministic
- Class P

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I-clicker question (frequency: AC)

Let $t(n)$ be a function, where $t(n) \geq n$.

Every 3-tape TM that runs in time O($t(n)$) can be simulated by a 1-tape TM that runs in time

A. **O($t(n)$)**

B. **O($t(n^2)$)**

C. **O($t(n^3)$)**

D. **O**$\left(\left(t(n)\right)^2\right)$

E. **Some 3-tape TMs can't be simulated by 1-tape TMs**

Complexity relationships between models: number of tapes

Theorem. Let $t(n)$ be a function, where $t(n) \geq n$.
Every $t(n)$ time multitape TM has
an equivalent $O\left(\left(t(n)\right)^2\right)$ time 1-tape TM.

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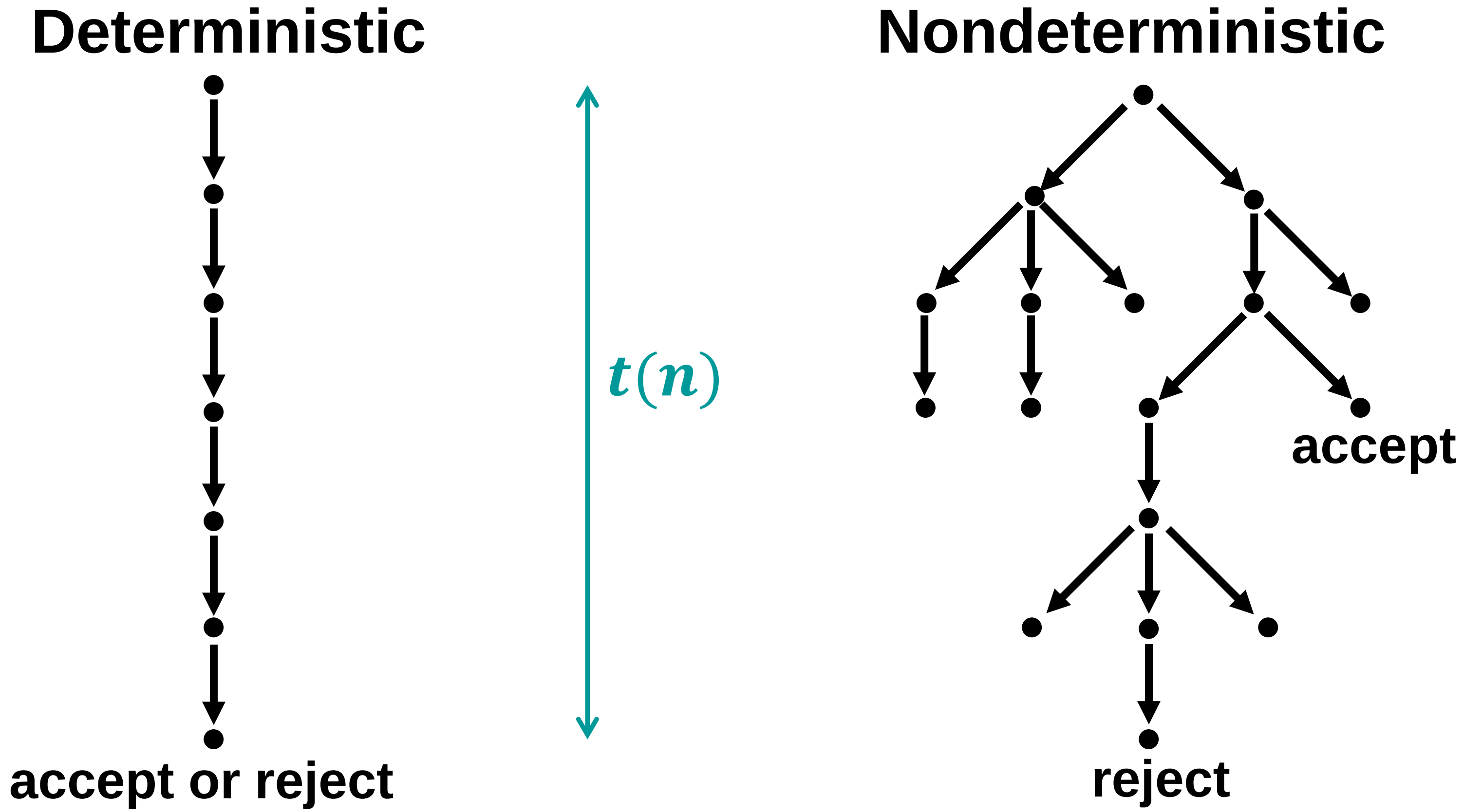
Time complexity of NTMs

The **running time** a nondeterministic **decider** N is $t(n)$ if on **all** inputs of length n, NTM N takes **at most $t(n)$** steps on the **longest** nondeterministic branch.

Time complexity of NTMs



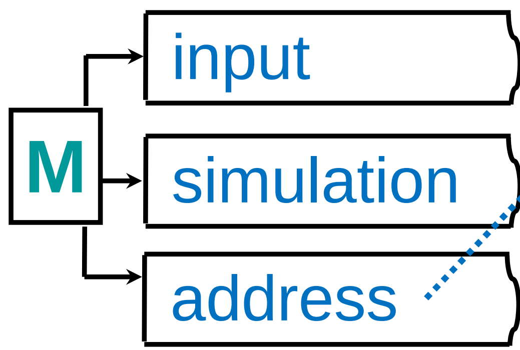
- Length of the longest computational branch, even if accepts before

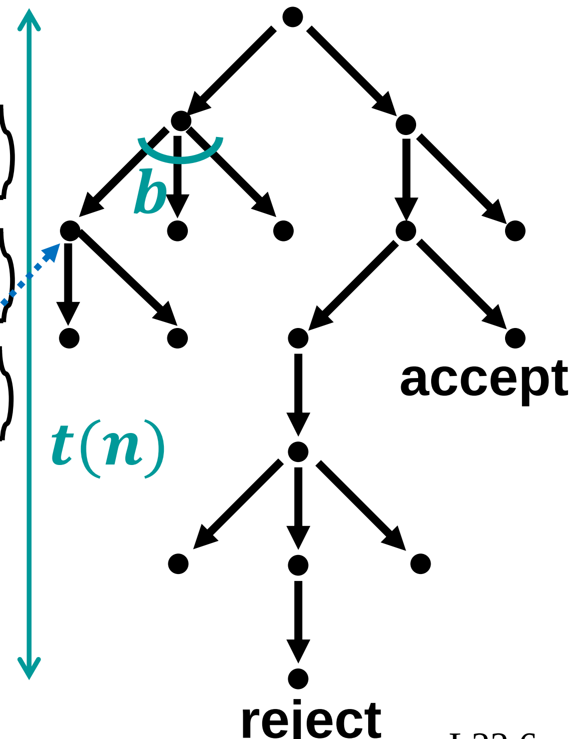
Complexity relationships between models: nondeterminism

Theorem. Let $t(n)$ be a function, where $t(n) \geq n$. Every $t(n)$ time nondeterministic TM has an equivalent $2^{O(t(n))}$ time 1-tape deterministic TM.

Proof: Simulate an NTM by a 3-tape TM.

- # of leaves $\leq \boldsymbol{b^{t(n)}}$
- # of nodes $\leq \boldsymbol{2b^{t(n)}}$





Time

- increment the address and

simulate from the root to a node: $\boldsymbol{O(t(n))}$

- Total: $\boldsymbol{O(t(n)b^{t(n)}) = 2^{O(t(n))}}$

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Theorem. Let $t(n)$ be a function, where $t(n) \geq n$.
Every $t(n)$ time nondeterministic TM has
an equivalent $2^{O(t(n))}$ time 1-tape deterministic TM.

Proof: So, a 3-tape TM can simulate an NTM in $2^{O(t(n))}$ time.

Converting to a 1-tape TM at most squares the running time:

$$\left(2^{O(t(n))}\right)^{\mathbf{2}} = 2^{O(\mathbf{2}\, t(n))} = 2^{O(t(n))}$$

Difference in time complexity

At most ***polynomial*** difference between ***all reasonable*** deterministic models.

At most ***exponential*** difference between deterministic and nondeterministic models.

The class P

P is the class of languages decidable in polynomial time on a ***deterministic*** 1-tape TM:

$$\boldsymbol{P} = \bigcup_{k} TIME(n^k).$$

- The same class even if we substitute another reasonable deterministic model.
- Roughly the class of problems realistically solvable on a computer.

Examples of languages in P

- PATH $=\{\langle G, s, t\rangle \mid G$ is a directed graph that has a directed path from s to $t\}$
- RELPRIME $= \{\langle x, y\rangle \mid x$ and y are relatively prime$\}$
- PRIMES $=\{x \mid x$ is a prime number$\}$ [2002]

- Every context-free language **(On the board)**

Recall: Chomsky Normal Form for CFGs

- Can have a rule $S \to \varepsilon$.
- All remaining rules are of the form

$A \to BC \qquad\qquad A, B, C \in V$

$A \to a \qquad\qquad a \in \Sigma$

- Cannot have S on the RHS of any rule.

Lemma. Any CFG can be converted into an equivalent CFG in Chomsky normal form.

Lemma. If G is in Chomsky normal form, any derivation of string w of length n in G has $2n - 1$ steps.

A decider for a CFL

- Let L be a CFL generated by a CFG G in CNF

M = `` On input $\langle w \rangle$, where w is a string:

1. **Let $n = |w|$.**
2. **Test all derivations with $2n - 1$ steps.**
3. **Accept if any derived w. O.w. reject.”**

- How long does it take? (Exponential time)
- Idea: use dynamic programming

(in the book)