Intro to Theory of Computation

CS 464

LECTURE 3

Last time:

- DFAs and NFAs
- Operations on languages

Today:

- Nondeterminism
- Equivalence of NFAs and DFAs
- Closure properties of regular languages

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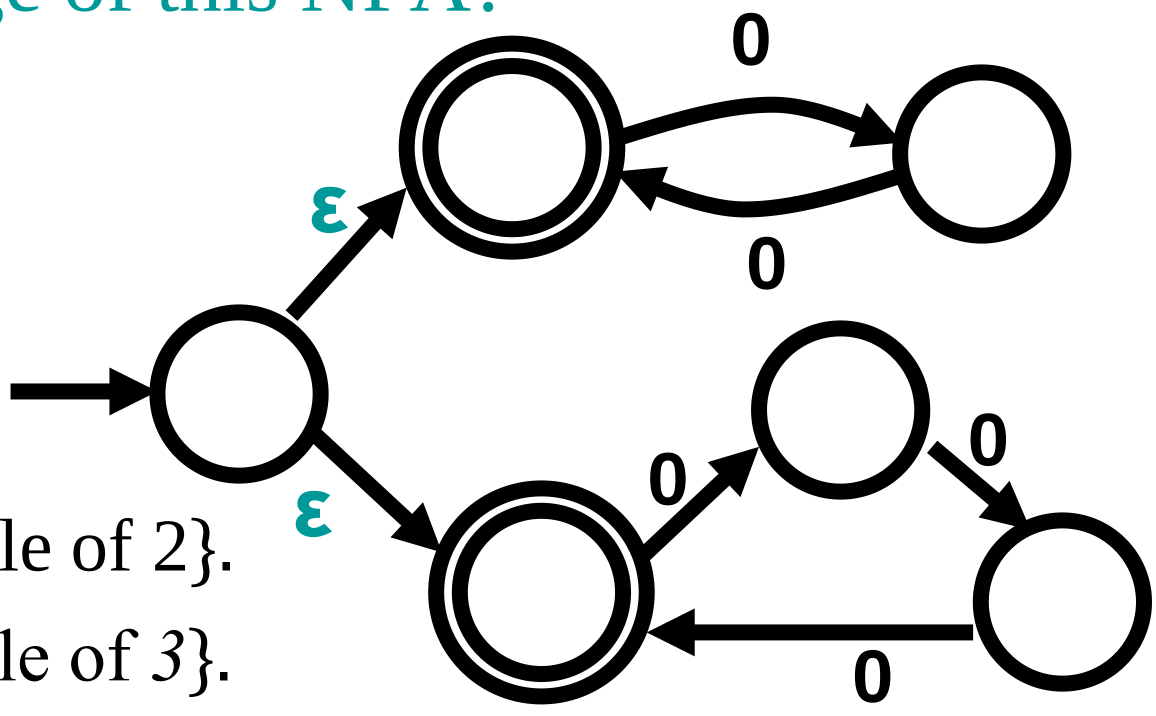
Nondeterminism

Nondeterministic Finite Automaton (NFA) accepts a string w if there is a way to make it reach an accept state on input w.

I-clicker problem (frequency: AC)

What is the language of this NFA?

(0^k means $\underbrace{00\ldots0}_{k}$)



A. $\{0^k \mid k$ is a multiple of 2$\}$.

B. $\{0^k \mid k$ is a multiple of 3$\}$.

C. $\{0^k \mid k$ is a multiple of 6$\}$.

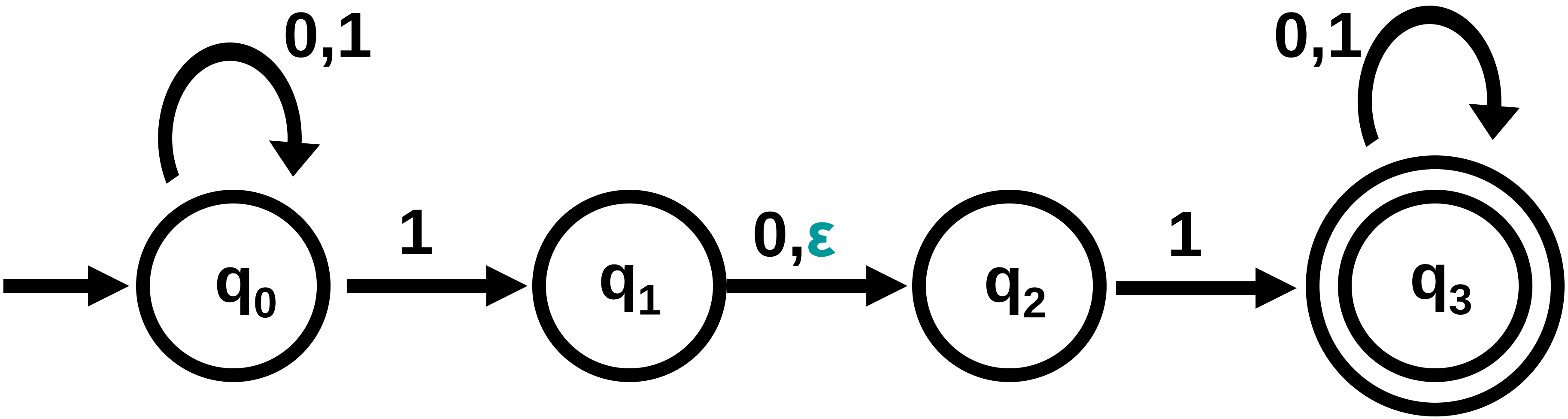
D. $\{0^k \mid k$ is a multiple of 2 or 3$\}$.

E. None of the above.

Formal Definition

- An ***NFA*** is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$
  - Q is the set of states
  - Σ is the alphabet
  - **$\delta : Q \times \Sigma_\varepsilon \to P(Q)$ is the transition function**
  - $q_0 \in Q$ is the start state
  - $F \subseteq Q$ is the set of accept states
- **$P(Q)$ is the set of subsets of Q and $\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$**
- **M** ***accepts*** a string **w** if **there is** a path from q_0 to an accept state that **w** follows.

Example



$N = (Q, \Sigma, \delta, q_0, F)$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0,1\}$

$F = \{q_3\}$

$\delta(q_0,0) = \{q_0\}$

$\delta(q_0,1) = \{q_0, q_1\}$

$\delta(q_1,\varepsilon) = \{q_1,q_2\}$

$\delta(q_2,0) = \varnothing$

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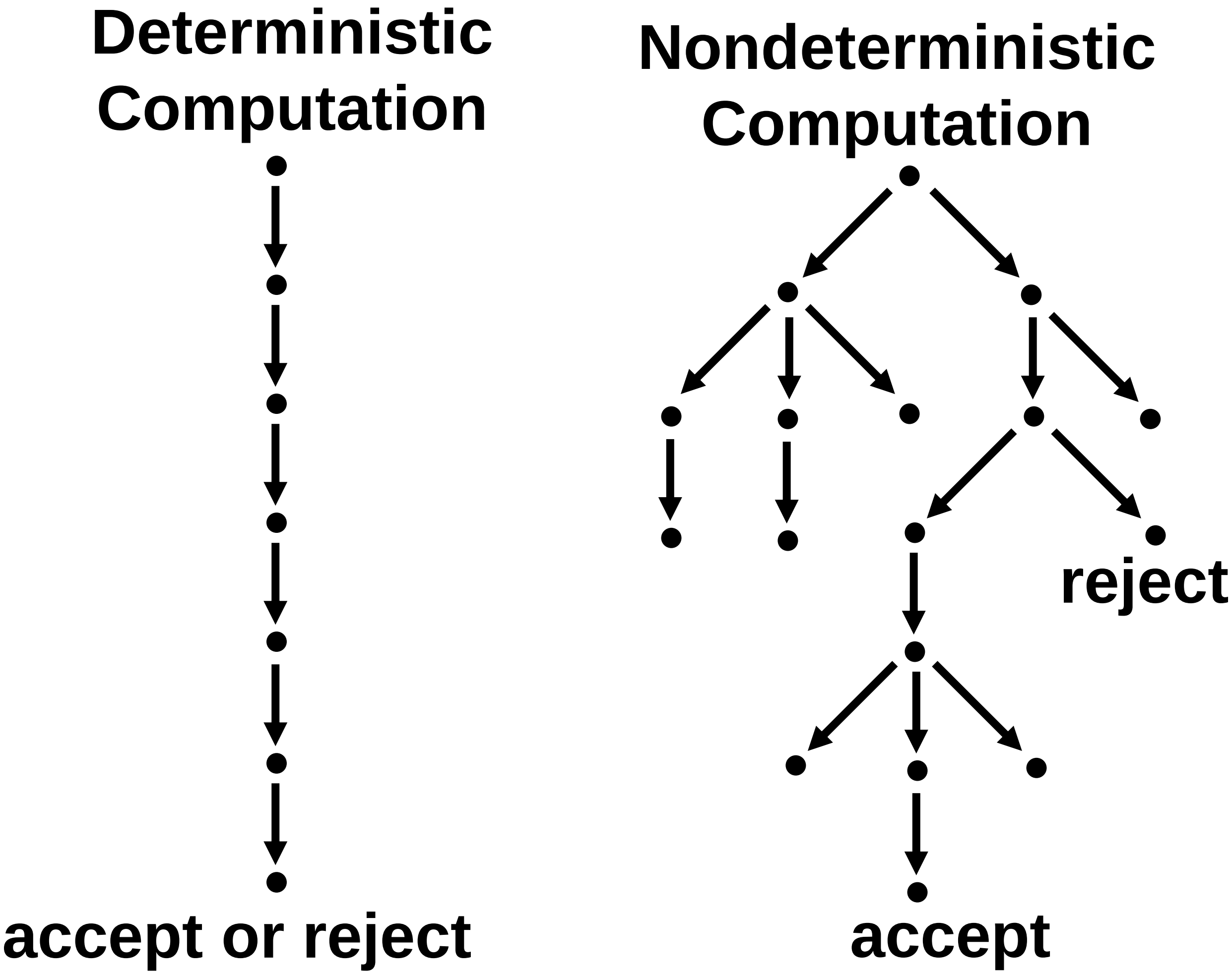
Nondeterminism

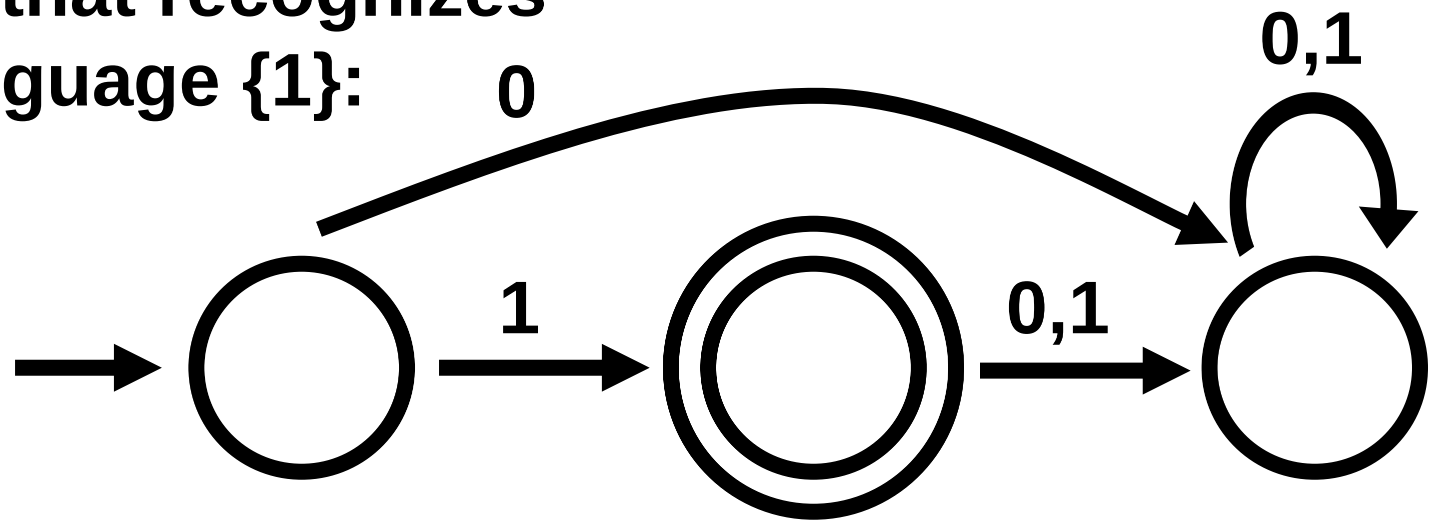


Ways to think about nondeterminism

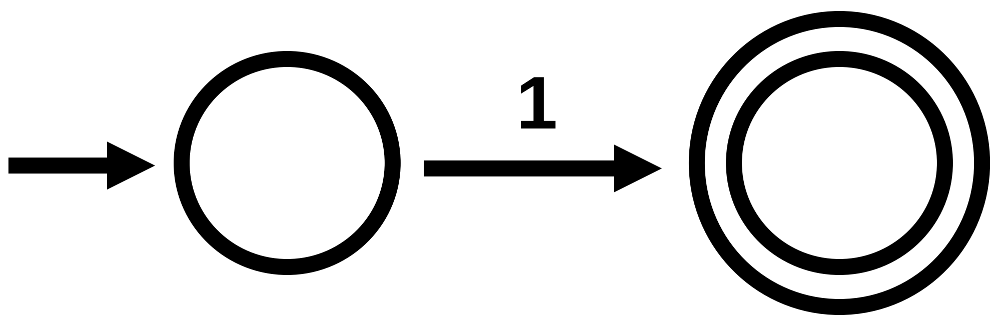
- **parallel computation**
- **tree of possible computations**
- **guessing and verifying the "right" choice**

NFAs ARE SIMPLER THAN DFAs

A DFA that recognizes
the language {1}:



An NFA that recognizes the language {1}:



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A DFA recognizing {1}

Theorem. Every DFA for language {1} must have at least 3 states.

Proof:

Equivalence of NFAs & DFAs

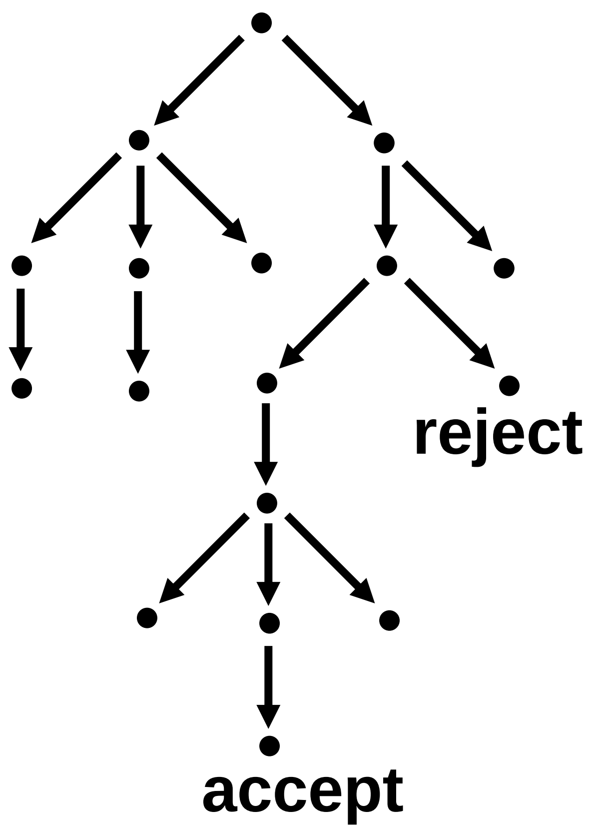
Theorem. Every NFA has an equivalent DFA.

Corollary: A language is regular iff
it is recognized by an NFA.

NFA to DFA Conversion

Input: $N = (Q, \Sigma, \delta, q_0, F)$

Output: $M = (Q', \Sigma, \delta', q_0', F')$



Intuition: Do the computation in parallel, maintaining the set of states where all threads are.

Idea:

$$Q' = P(Q)$$

NFA to DFA Conversion

Input: $N = (Q, \Sigma, \delta, q_0, F)$

Output: $M = (Q', \Sigma, \delta', q_0', F')$

For $R \subseteq Q$, let $E(R)$ be the set of states reachable by ε-transitions from the states in R.

$Q' = P(Q)$

$\delta' : Q' \times \Sigma \rightarrow Q'$

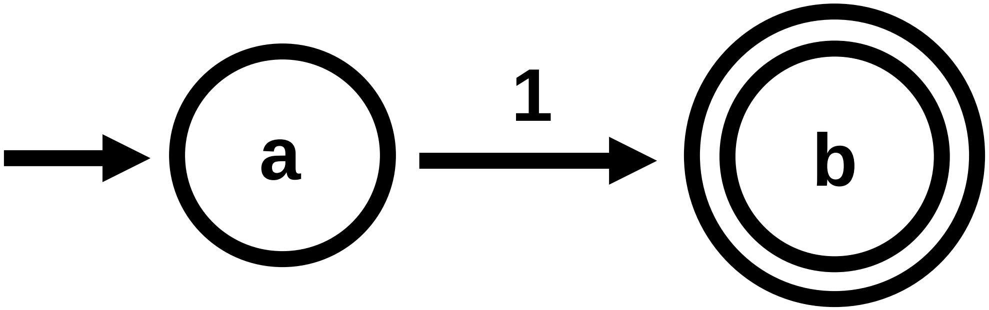
$\delta'(R,\sigma) = \bigcup_{r \in R} (\delta(r,\sigma))$ for all $R \subseteq Q$ and $\sigma \in \Sigma$.

$q_0' = (\{q_0\})$

$F' = \{ R \in Q' \mid R \text{ contains some accept state of } N\}$

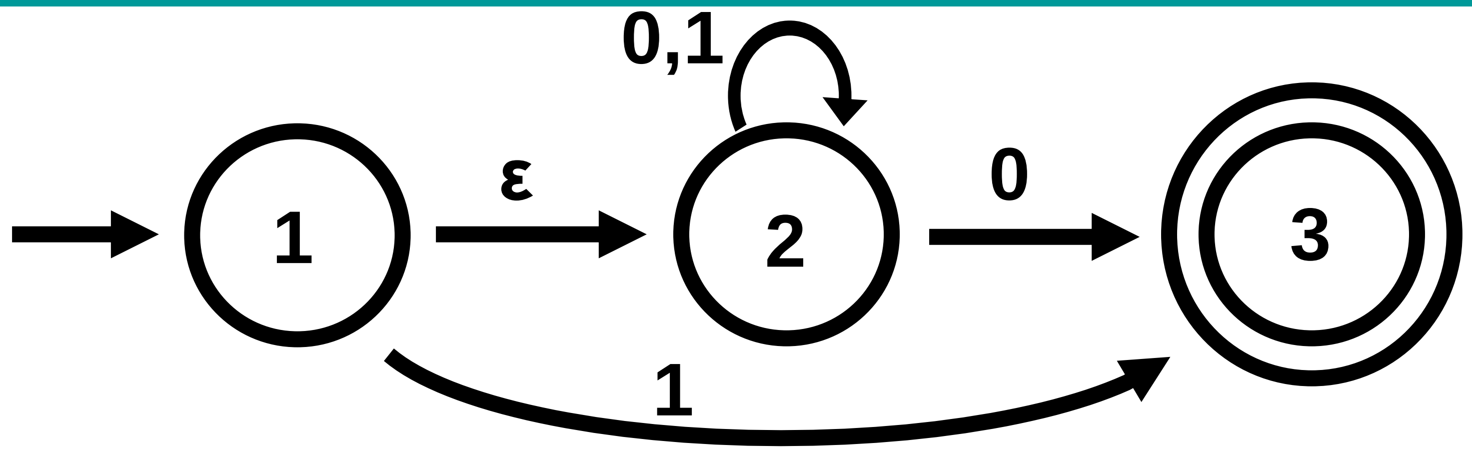
Example: NFA to DFA

1)



Examples NFA to DFA

2)



Regular Operations on languages

Complement: $\neg A = \{ w \mid w \notin A \}$

Union: $A \cup B = \{ w \mid w \in A \text{ or } w \in B \}$

Intersection: $A \cap B = \{ w \mid w \in A \text{ and } w \in B \}$

Reverse: $A^R = \{ w_1 \ldots w_k \mid w_k \ldots w_1 \in A \}$

Concatenation: $A \circ B = \{ vw \mid v \in A \text{ and } w \in B \}$

Star: $A^* = \{ w_1 \ldots w_k \mid k \geq 0 \text{ and each } w_i \in A \}$

Closure properties of the class of regular languages

THEOREM. The class of regular languages is **closed** under all 6 operations.

If A and B are regular, applying any of these operation yields a regular language.

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I-clicker problem (frequency: AC)

Let L be the set of words in English.

Then $L \cap L^R$ is

A. The set of English words in alphabetical order, followed the same words in reverse alphabetical order.

B. {*w* | *w* is an English word or an English word written backwords}.

C. {*w* | *w* is an English word that is a palindrome}.

D. None of the above.

Palindromes

A palindrome is a word or a phrase that reads the same forward and backward.

Examples

- mom
- madam
- Never odd or even.
- Stressed? No tips? Spit on desserts!

I-clicker problem (frequency: AC)

Let L be the set of words in English.

Then $L \cap L^R$ is

A. The set of English words in alphabetical order, followed the same words in reverse alphabetical order.

B. $\{w \mid w$ is an English word or an English word written backwords$\}$.

C. $\{w \mid w$ is an English word that is a palindrome$\}$.

D. None of the above.