Intro to Theory of Computation

Lecture 3

Last time:
• DFAs and NFAs
• Operations on languages

Today:
• Nondeterminism
• Equivalence of NFAs and DFAs
• Closure properties of regular languages

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Nondeterministic Finite Automaton (NFA) accepts a string $w$ if there is a way to make it reach an accept state on input $w$. 
I-caller problem (frequency: AC)

What is the language of this NFA?

(\(0^k\) means \(00...0\))

A. \(\{0^k \mid k \text{ is a multiple of 2}\}\).
B. \(\{0^k \mid k \text{ is a multiple of 3}\}\).
C. \(\{0^k \mid k \text{ is a multiple of 6}\}\).
D. \(\{0^k \mid k \text{ is a multiple of 2 or 3}\}\).
E. None of the above.
Formal Definition

- An **NFA** is a 5-tuple \( M = (Q, \Sigma, \delta, q_0, F) \)
  - \( Q \) is the set of states
  - \( \Sigma \) is the alphabet
  - \( \delta : Q \times \Sigma \varepsilon \rightarrow P(Q) \) is the transition function
  - \( q_0 \in Q \) is the start state
  - \( F \subseteq Q \) is the set of accept states

- \( P(Q) \) is the set of subsets of \( Q \) and \( \Sigma \varepsilon = \Sigma \cup \{ \varepsilon \} \)

- \( M \) **accepts** a string \( w \) if there is a path from \( q_0 \) to an accept state that \( w \) follows.
Example

\[ N = (Q, \Sigma, \delta, q_0, F) \]

\[ Q = \{q_0, q_1, q_2, q_3\} \]

\[ \Sigma = \{0,1\} \]

\[ F = \{q_3\} \]

\[ \delta(q_0,0) = \{q_0\} \]

\[ \delta(q_0,1) = \{q_0, q_1\} \]

\[ \delta(q_1,\varepsilon) = \{q_1, q_2\} \]

\[ \delta(q_2,0) = \emptyset \]
Nondeterminism

Ways to think about nondeterminism

• parallel computation
• tree of possible computations
• guessing and verifying the “right” choice
NFAs ARE SIMPLER THAN DFAs

A DFA that recognizes the language \{1\}:

An NFA that recognizes the language \{1\}:
A DFA recognizing \{1\}

**Theorem.** Every DFA for language \{1\} must have at least 3 states.

**Proof:**
Theorem. Every NFA has an equivalent DFA.

Corollary: A language is regular iff it is recognized by an NFA.
Input: \( N = (Q, \Sigma, \delta, q_0, F) \)
Output: \( M = (Q', \Sigma, \delta', q_0', F') \)

**Intuition:** Do the computation in parallel, maintaining the set of states where all threads are.

**Idea:**
\[ Q' = P(Q) \]
NFA to DFA Conversion

Input: \( N = (Q, \Sigma, \delta, q_0, F) \)
Output: \( M = (Q', \Sigma, \delta', q_0', F') \)

For \( R \subseteq Q \), let \( E(R) \) be the set of states reachable by \( \varepsilon \)-transitions from the states in \( R \).

\[
Q' = P(Q) \\
\delta' : Q' \times \Sigma \to Q' \\
\delta'(R, \sigma) = \bigcup_{r \in R} (\delta(r, \sigma)) \text{ for all } R \subseteq Q \text{ and } \sigma \in \Sigma. \\
q_0' = (\{q_0\}) \\
F' = \{ R \in Q' \mid R \text{ contains some accept state of } N \}
Example: NFA to DFA

1)
Examples NFA to DFA

2)

\[ L_3 \]

\[ L_3 \]
Regular Operations on languages

Complement: \( \overline{A} = \{ w \mid w \notin A \} \)

Union: \( A \cup B = \{ w \mid w \in A \text{ or } w \in B \} \)

Intersection: \( A \cap B = \{ w \mid w \in A \text{ and } w \in B \} \)

Reverse: \( A^R = \{ w_1 \ldots w_k \mid w_k \ldots w_1 \in A \} \)

Concatenation: \( A \circ B = \{ vw \mid v \in A \text{ and } w \in B \} \)

Star: \( A^* = \{ w_1 \ldots w_k \mid k \geq 0 \text{ and each } w_i \in A \} \)
Closure properties of the class of regular languages

**THEOREM.** The class of regular languages is **closed** under all 6 operations.

If A and B are regular, applying any of these operation yields a regular language.
Let $L$ be the set of words in English.

Then $L \cap L^R$ is

A. The set of English words in alphabetical order, followed the same words in reverse alphabetical order.

B. $\{w \mid w$ is an English word or an English word written backwards$\}$.

C. $\{w \mid w$ is an English word that is a palindrome$\}$.

D. None of the above.
A **palindrome** is a word or a phrase that reads the same forward and backward.

**Examples**

- mom
- madam
- Never odd or even.
- Stressed? No tips? Spit on desserts!
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Then $L \cap L^R$ is
A. The set of English words in alphabetical order, followed the same words in reverse alphabetical order.
B. $\{w \mid w$ is an English word or an English word written backwards$\}$.
C. $\{w \mid w$ is an English word that is a palindrome$\}$.
D. None of the above.