

Homework 3 – Due Thursday, February 4, 2016 before the lecture

Please refer to the general information handout for the full homework policy.

Reminder Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to the course staff if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Exercises Please practice on exercises and solved problems in Chapter 2. The material they cover may appear on exams.

Problems

1. (Pumping lemma and irregular languages)

- (a) Sipser, 1.30.
- (b) Use the pumping lemma to prove that the following language over the alphabet $\Sigma = \{\#, 1\}$ is not regular: $L_1 = \{w \mid w = 1^t \# x_1 \# x_2 \# \dots \# x_k, \text{ for } t \geq k \geq 0 \text{ and each } x_i \in 1^*\}$.
- (c) Use closure properties to prove that the language $L_2 = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$ is not regular.
- (d) Recall that we showed in lecture that L_2 satisfies the pumping lemma. Explain why this fact and part (c) do not contradict the pumping lemma?

2. (PDAs) Draw state diagrams of PDAs (with as few states as you can) that recognize the following languages. Write an algorithmic description for your PDAs.

- (a) Let $\Sigma_2 = \{[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}], [\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}], [\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}], [\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}]\}$.
 $L_3 = \{w \in \Sigma_2^* \mid \text{the bottom row of } w \text{ is the reverse of the top row of } w\}$.
- (b) L_4 is the collection of all strings that contain at least one 1 in their second half (if the string is of odd length, we exclude the middle symbol to construct the second “half”). In other words, $L_4 = \{uv \mid u \in \Sigma^*, v \in \Sigma^* 1 \Sigma^* \text{ and } |u| \geq |v|\}$.

3. (BALANCE operation) Given a string w over alphabet $\Sigma = \{0, 1\}$, define

$$\text{BALANCE}(w) = \begin{cases} (01)^x & \text{if } x = y, \\ (01)^y 0^{x-y} & \text{if } x > y, \\ (01)^x 1^{y-x} & \text{if } x < y, \end{cases}$$

where x is the number of 0s in w and y is the number of 1s in w .

For example, $\text{BALANCE}(000111) = 010101$ and $\text{BALANCE}(0100000111) = 0101010100$. Note that w and $\text{BALANCE}(w)$ have the same number of 0s and 1s, but in the beginning of the string $\text{BALANCE}(w)$, the 0 and 1 characters alternate until one of them runs out.

Define the corresponding operation on languages:

$$\text{BALANCE}(L) = \{\text{BALANCE}(w) \mid w \in L\}.$$

- (a) Give $\text{BALANCE}(L)$ for the language $L = \{00011, 000, 11110, 101, 110\}$.
- (b) Give a regular expression for the language $\text{BALANCE}(0^*1^*)$.
- (c) Show that the class of regular languages is *not* closed under the BALANCE operation.
- (d) Prove that if A is regular then there is a PDA that recognizes $\text{BALANCE}(A)$. An algorithmic description of your PDA is sufficient.

Hint: Make sure that your PDA checks two conditions for every string it accepts: (1) that it is of the form specified by your regular expression; (2) the 0's and 1's in the string can be reordered to obtain a string in A .

- 4* (**Optional, no collaboration is allowed, please hand in your solution on a separate sheet of paper**) Give a PDA that recognizes the following language over the alphabet $\Sigma = \{0, 1\}$:
 $L = \{xyz \mid x, z \in \Sigma^* \text{ and } y \in \Sigma^*1\Sigma^*, \text{ where } |x| = |z| \geq |y|\}$.