

Homework 0 – Due Thursday, January 14, 2016 before the lecture

Please refer to the general information handout for the full homework policy and options.

Page limit You can submit **at most** 1 page per problem, even if the problem has multiple parts. If you submit a longer solution for some problem, only the first page will be graded.

Exercises Please practice on exercises and solved problems in Sipser, Chapter 0 and odd-numbered exercises in Hammack, Chapters 1,2, 4–6 (solutions are provided on the book’s web page). The material they cover may appear on exams.

Problems

0. (**0 points**) The following steps are required to get you started in the course.

- (a) Sign up on piazza at piazza.com/psu/spring2016/cmpsc464.
- (b) Register your clicker at: clickers.psu.edu.
- (c) Read and sign the Collaboration and Honesty Policy. We will be able to grade your homework only after you hand this in.
- (d) Check out the following links and resources:
 - i. course webpage: <http://www.cse.psu.edu/~sofya/cmpsc464/>;
 - ii. supplementary textbook to review proof techniques:
Richard Hammack. *Book of Proof*. <http://www.people.vcu.edu/~rhammack/BookOfProof/>
 - iii. practice with automata: <http://automatatutor.com/> and <http://jflap.org/>.

1. (**Angel problem: logic and sets review, 10 points**) Enter the answers to these questions on Angel. (On Angel, all parts are phrased as multiple-choice or fill-in-the-blank, so that they can be graded automatically.)

Negate the following statements:

- (a) The first DFA has an even number of states, but the second DFA does not.
- (b) Whenever I have to choose between two evils, I choose the one I haven’t tried before.

Write the contrapositive of the following statement:

- (c) Your homework assignments is not graded if you have not handed in a signed copy of the Collaboration policy.

Answer the following questions about basic set operations:

- (d) If $A = \{x, y, z\}$ and $B = \{x, y\}$, what is $A \times B$? What is its size?
- (e) If $A = \{x, y, z\}$, what is the power set of A ? What is its size?

2. (**Proof techniques review, 10 points**) Type (preferred) or write the answer to this problem on a single sheet of paper. Specify your **name**, **section** and **collaborators** (or state “Collaborators: none”) at the top of the sheet.

- (a) (**Induction.**) Find the error in the following proof that all students at Penn State have the same GPA. Indicate the **first sentence in the proof** that is incorrect. (There must be an incorrect statement in the proof, since the claim is wrong.)

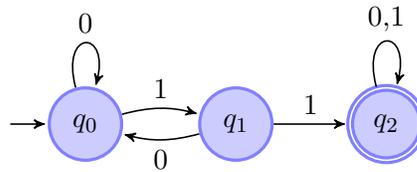
CLAIM: In every set of n Penn State students, all students have the same GPA.

PROOF: By induction on n .

Base case: $n = 1$. In every set containing just one Penn State student, all students clearly have the same GPA.

Induction step: For $k \geq 1$ assume that the claim holds for $n = k$ and prove that it holds for $n = k + 1$. Take any set S of $k + 1$ Penn State students. Remove one student from this set to obtain the set S_1 with k students. By the induction hypothesis, all students in S_1 have the same GPA. Now replace the removed student and remove a different one to obtain the set S_2 . By the same argument, all students in S_2 have the same GPA. Therefore, all the students in S must have the same GPA.

- (b) (**Direct proof.**) Give a direct proof that the following automaton accepts an infinite number of strings.



- (c) (**Contradiction.**) Prove the same thing as in part (b) by contradiction.