Homework 0 – Due Thursday, January 14, 2016 before the lecture

Please refer to the general information handout for the full homework policy and options.

Page limit  You can submit at most 1 page per problem, even if the problem has multiple parts. If you submit a longer solution for some problem, only the first page will be graded.

Exercises  Please practice on exercises and solved problems in Sipser, Chapter 0 and odd-numbered exercises in Hammack, Chapters 1, 2, 4–6 (solutions are provided on the book’s web page). The material they cover may appear on exams.

Problems

0. (0 points) The following steps are required to get you started in the course.

(a) Sign up on piazza at piazza.com/psu/spring2016/cmpsc464
(b) Register your clicker at: clickers.psu.edu
(c) Read and sign the Collaboration and Honesty Policy. We will be able to grade your homework only after you hand this in.
(d) Check out the following links and resources:
   i. course webpage: http://www.cse.psu.edu/~sofya/cmpsc464/
   ii. supplementary textbook to review proof techniques:
   iii. practice with automata: http://automatatutor.com/ and http://jflap.org/

1. (Angel problem: logic and sets review, 10 points) Enter the answers to these questions on Angel. (On Angel, all parts are phrased as multiple-choice or fill-in-the-blank, so that they can be graded automatically.)

Negate the following statements:

(a) The first DFA has an even number of states, but the second DFA does not.
(b) Whenever I have to choose between two evils, I choose the one I haven’t tried before.

Write the contrapositive of the following statement:

(c) Your homework assignments is not graded if you have not handed in a signed copy of the Collaboration policy.

Answer the following questions about basic set operations:

(d) If \( A = \{x, y, z\} \) and \( B = \{x, y\} \), what is \( A \times B \)? What is its size?
(e) If \( A = \{x, y, z\} \), what is the power set of \( A \)? What is its size?

2. (Proof techniques review, 10 points) Type (preferred) or write the answer to this problem on a single sheet of paper. Specify your name, section and collaborators (or state “Collaborators: none”) at the top of the sheet.
(a) (Induction.) Find the error in the following proof that all students at Penn State have the same GPA. Indicate the first sentence in the proof that is incorrect. (There must be an incorrect statement in the proof, since the claim is wrong.)

Claim: In every set of \( n \) Penn State students, all students have the same GPA.

Proof: By induction on \( n \).

Base case: \( n = 1 \). In every set containing just one Penn State student, all students clearly have the same GPA.

Induction step: For \( k \geq 1 \) assume that the claim holds for \( n = k \) and prove that it holds for \( n = k + 1 \). Take any set \( S \) of \( k + 1 \) Penn State students. Remove one student from this set to obtain the set \( S_1 \) with \( k \) students. By the induction hypothesis, all students in \( S_1 \) have the same GPA. Now replace the removed student and remove a different one to obtain the set \( S_2 \). By the same argument, all students in \( S_2 \) have the same GPA. Therefore, all the students in \( S \) must have the same GPA.

(b) (Direct proof.) Give a direct proof that the following automaton accepts an infinite number of strings.

(c) (Contradiction.) Prove the same thing as in part (b) by contradiction.