Homework 9 – Due Wednesday, November 14, 2007

Please refer to the general information handout for the full homework policy and options.

Reminders

- Your solutions are due *before* the lecture. Late homework will not be accepted.
- Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to a member of the course staff if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.
- To facilitate grading, please write down your solution to each problem on a separate sheet of paper. Make sure to include all identifying information and your collaborators on each sheet. Your solutions to different problems will be graded separately, possibly by different people, and returned to you independently of each other.
- *For all problems where you are asked to design an algorithm, do not forget to prove correctness and analyze your algorithm's time and space complexity.*

General guidelines for reductions. Model your solutions on the reduction of Maximum Matching to Maximum Flow given in class. To reduce problem B to problem A:

1. Explain how to transform an instance $\mathcal{I}_B$ of B into an instance $\mathcal{I}_A$ of A.
2. Explain how to transform a solution $\mathcal{S}_A$ for $\mathcal{I}_A$ into a solution $\mathcal{S}_B$ for $\mathcal{I}_B$.
3. (**large fraction of the points**) Prove that $\mathcal{S}_B$ is a correct solution for $\mathcal{I}_B$, provided that $\mathcal{S}_A$ is a correct solution for $\mathcal{I}_A$. In case of optimization problems, it usually involves proving that the value of $\mathcal{S}_A$ is equal to the value of $\mathcal{S}_B$. (Often, it is easier to prove $\geq$ and $\leq$ separately.)
4. Analyze the efficiency of the resulting algorithm for problem B that uses your reduction and the most suitable algorithm for problem A that we studied. Make sure that the running time is expressed in terms of the length of $\mathcal{I}_B$, not $\mathcal{I}_A$.

Exercises These should not be handed in, but the material they cover may appear on exams:

1. (a) Reduce the maximum flow problem for a network with several source nodes $(s_1, \ldots, s_k)$ and several sink nodes $(t_1, \ldots, t_\ell)$ into the single-source single-sink maximum flow problem.
   (b) Some networks have capacity constraints on the flow amounts that can flow through their intermediate vertices. Explain how the maximum flow problem for such a network can be reduced to Maximum Flow with edge capacity constraints only.
2. As usual, read, solve and check your answers on the solved exercises in Chapter 7. They might be helpful for some of the homework problems.

Problems to be handed in

1. (**Perfect matching in a k-regular graph**) You are given a bipartite graph G with vertex sets L and R.

   (a) Prove that if every node in L and R has degree $k \geq 1$ then G has a perfect matching. Deduce (i.e., prove assuming the previous statement) that such a graph has k disjoint perfect matchings.

   (b) Show that the first statement you are asked to prove in part (a) is false when "degree k" is replaced with "degree at least k".

2. (**Clients and base stations**) Chapter 7, problem 7. Your running time should be polynomial in n and k and independent of r and L, assuming that arithmetic operations on integers can be performed in constant time.

3. (**Blood supply**) Chapter 7, problem 8.

4. (**Dining Problem**) Several families go out to dinner together. To increase their social interaction, they would like to sit at tables so that no two members of the same family are at the same table. Show how to find a seating arrangement that meets this objective (or prove that no such arrangement exists) by reducing this problem to Maximum Flow. Assume that the dinner contingent has p families, that the ith family has a_i members, that q tables are available and up to b_j people can be seated at table j.