
Homework 6 – Due Wednesday, October 17, 2007

Please refer to the general information handout for the full homework policy and options.

Reminders

- Your solutions are due *before* the lecture. Late homework will not be accepted.
- Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to a member of the course staff if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.
- To facilitate grading, please write down your solution to each problem on a separate sheet of paper. Make sure to include all identifying information and your collaborators on each sheet. Your solutions to different problems will be graded separately, possibly by different people, and returned to you independently of each other.
- *For all problems where you are asked to design an algorithm, do not forget to prove correctness and analyze your algorithm's time and space complexity.*

Exercises These should not be handed in, but the material they cover may appear on exams:

1. Compute the DFT of the vector $(0, 1, 2, 3)$.
2. Multiply the polynomials $A(x) = 7x^3 - x^2 + x - 10$ and $B(x) = 8x^3 - 6x + 3$ using the FFT scheme. Verify your answer by directly multiplying the polynomials using the straightforward algorithm that runs in $\Theta(n^2)$ time.
3. You might want to solve a few problems from Chapter 6 to master dynamic programming.

General hint for designing divide-and-conquer algorithms: When the target running time is given, think which recurrence would yield this running time. Try to divide your input accordingly and see if you can combine the results of the recursive calls in the required time.

Problems to be handed in

1. **(Polynomials)**
 - (a) Consider two sets A and B , each containing n integers in the range from 0 to $10n$. The **Cartesian sum** of A and B , defined by

$$C = \{x + y : x \in A \text{ and } y \in B\}.$$

Note that the integers in C are in the range from 0 to $20n$. We want to find the elements of C and the number of times each element of C is realized as a sum of elements in A and B . Show that the problem can be solved in $O(n \log n)$ time. (*Hint:* Represent A and B as polynomials of degree at most $10n$.)

- (b) Given a list of values z_0, z_1, \dots, z_{n-1} (possibly with repetitions), show how to find the coefficients of a polynomial $P(x)$ of degree n that has zeros only at z_0, z_1, \dots, z_{n-1} . Your algorithm should run in time $O(n \log^2 n)$. (*Hints:* The polynomial $P(x)$ has a zero at z_j if and only if $P(x)$ is a multiple of $(x - z_j)$. Use divide and conquer.)
2. (**High and low stress jobs**) Chapter 6, problem 2.
3. (**Longest path in an ordered graph**) Chapter 6, problem 3.