Intro to Theory of Computation

LECTURE 2

Theory of Computation

- Finite Automata
- Operations on languages
- Nondeterminism

Homework 0 due
Homework 1 handed out

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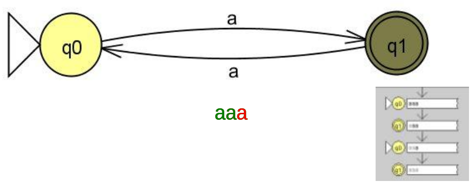
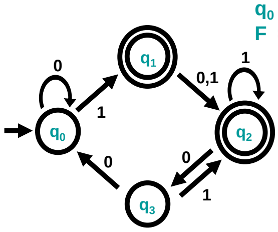
JFLAP http://www.cs.duke.edu/csed/jflap/



Formal definition of FA

$M = (Q, \Sigma, \delta, q_0, F)$ where

$Q = \{q_0, q_1, q_2, q_3\}$
$\Sigma = \{0,1\}$
$\delta : Q \times \Sigma \rightarrow Q$ transition function*
$q_0 \in Q$ is start state
$F = \{q_1, q_2\} \subseteq Q$ accept states



* δ	0	1
q_0		
q_1		
q_2		
q_3		

Language of FA

The **language of M**, **L(M),** is the set of strings that M accepts

A language is regular if it is recognized by a finite automaton

L = { w | w contains 001} is regular

L = { w | w has an even number of 1s} is regular

Regular languages

Many interesting programs accept regular languages

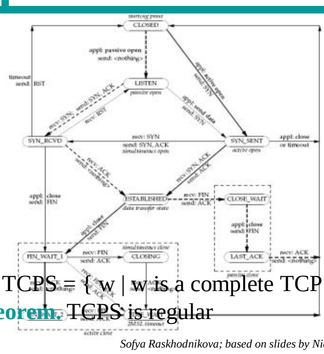
NETWORK PROTOCOLS

COMPILERS

GENETIC TESTING

ARITHMETIC

INTERNET TRANSMISSION CONTROL PROTOCOL



Let TCPS = { w | w is a complete TCP Session}
Theorem. TCPS is regular

CS 464 Compilers

COMMENTS :
Are delimited by /* */
Cannot have NESTED /* */
Must be CLOSED by */
*/ is ILLEGAL outside a comment

COMMENTS = {strings with legal comments}

Theorem. **COMMENTS** is regular.

CS 464 Genetic testing

DNA SEQUENCES are strings over the alphabet {A,C,G,T}

GENES are special substrings

A GENETIC TEST searches a DNA SEQUENCE for a GENE

Theorem. Every GENETIC TEST is regular.

CS 464 Arithmetic

LET $\Sigma_3 = \left\{ \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix} \right\}$

- A string over Σ_3 has three ROWS
- Each ROW $b_0b_1b_2\dots b_N$ represents the integer $b_0 + 2b_1 + \dots + 2^N b_N$.
- Let ADD = {S | $\text{ROW}_1 + \text{ROW}_2 = \text{ROW}_3$ }

Theorem. ADD is regular.

CS 464 Regular Operations on languages

Complement: $\neg A = \{ w \mid w \notin A \}$

Union: $A \cup B = \{ w \mid w \in A \text{ or } w \in B \}$

Intersection: $A \cap B = \{ w \mid w \in A \text{ and } w \in B \}$

Reverse: $A^R = \{ w_1 \dots w_k \mid w_k \dots w_1 \in A \}$

Concatenation: $A \circ B = \{ vw \mid v \in A \text{ and } w \in B \}$

Star: $A^* = \{ w_1 \dots w_k \mid k \geq 0 \text{ and each } w_i \in A \}$

CS 464 Closure properties of the class of regular languages

THEOREM. The class of regular languages is **closed** under all 6 operations.

If A and B are regular, applying any of these operation yields a regular language.

CS 464 Closure properties

Complement: $\neg A = \{ w \mid w \notin A \}$

Union: $A \cup B = \{ w \mid w \in A \text{ or } w \in B \}$

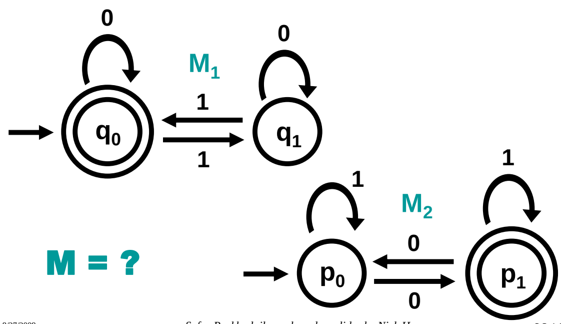
Closure under union

Theorem. The union of two regular languages is also a regular language

Proof: Let
$M_1 = (Q_1, \Sigma, \delta_1, q_0^1, F_1)$ be finite automaton for L_1
and
$M_2 = (Q_2, \Sigma, \delta_2, q_0^2, F_2)$ be finite automaton for L_2

Construct a finite automaton $M = (Q, \Sigma, \delta, q_0, F)$ that recognizes $L = L_1 \cup L_2$

Example



Proof (continued)

Idea: Run both M_1 and M_2 at the same time!

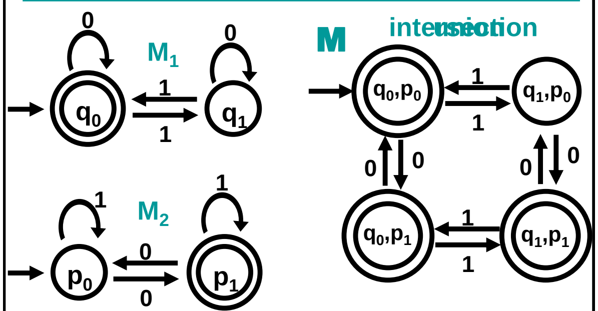
Q = pairs of states, one from M_1 and one from M_2
$= \{ (q_1, q_2) \mid q_1 \in Q_1 \text{ and } q_2 \in Q_2 \}$
$= Q_1 \times Q_2$

$q_0 = (q_0^1, q_0^2)$

$F = \{ (q_1, q_2) \mid q_1 \in F_1 \text{ or } q_2 \in F_2 \}$

$\delta((q_1,q_2), \sigma) = (\delta_1(q_1, \sigma), \delta_2(q_2, \sigma))$

Example (continued)



Closure properties

Complement: $\neg A = \{ w \mid w \notin A \}$

Union: $A \cup B = \{ w \mid w \in A \text{ or } w \in B \}$

Intersection: $A \cap B = \{ w \mid w \in A \text{ and } w \in B \}$

Reverse: $A^R = \{ w_1 \ldots w_k \mid w_k \ldots w_1 \in A \}$

Closure under reverse

Theorem. The reverse of a regular language is also regular language

Proof: Let L be a regular language and M be a finite automaton that recognizes it.

Construct a finite automaton M^R recognizing L^R

Idea: Define M^R as M with the arrows reversed
Swap start and accept states

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Closure under reverse

M^R IS NOT ALWAYS A DFA!

It may have many start states

Some states may have too many outgoing edges, or none

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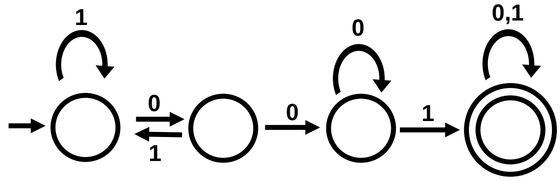
Example



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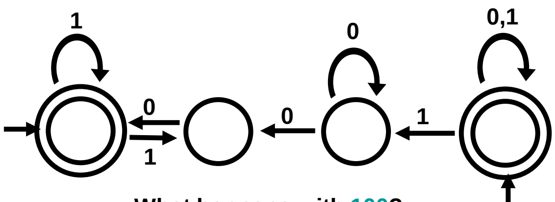
NONDETERMINISM



What happens with 100?

Nondeterministic Finite Automaton (NFA) accepts if there is a way to make it reach an accept state

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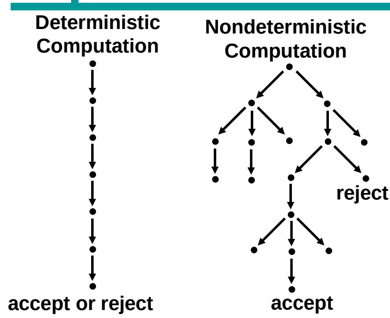
Nondeterminism



Ways to think about nondeterminism

- parallel computation
- tree of possible computations
- guessing the "right" choice

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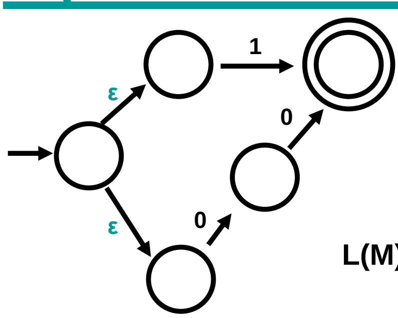
Example



L(M)={1,00}

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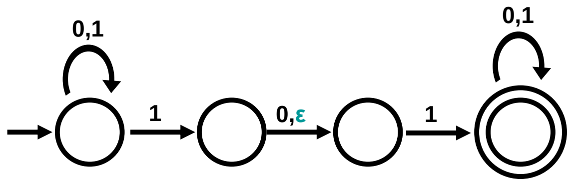
Example



L(M)={w | w contains 101 or 11}

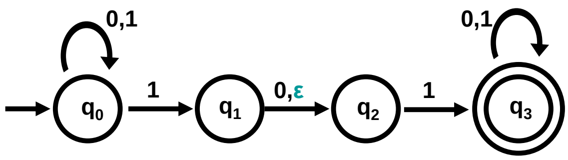
Formal Definition

An ***NFA*** is a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$

- Q is the set of states
- Σ is the alphabet
- $\delta : Q \times \Sigma_\varepsilon \to \mathcal{P}(Q)$ **is the transition function**
- $q_0 \in Q$ is the start state
- $F \subseteq Q$ is the set of accept states

$\mathcal{P}(Q)$ **is the set of subsets of Q and** $\Sigma_\varepsilon = \Sigma \cup \{\varepsilon\}$

Example



$N = (Q, \Sigma, \delta, q_0, F)$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0,1\}$

$F = \{q_3\}$

$\delta(q_0,1) = \{q_0, q_1\}$

$\delta(q_1,\varepsilon) = \{q_1,q_2\}$

$\delta(q_2,0) = \varnothing$