

## Homework 7 – Due Thursday, October 22, 2009 before the lecture

Please refer to the general information handout for the full homework policy and options. This homework contains 3 mandatory and one optional problem, worth 10 points each. *Your solution to each problem should be handed in on a separate sheet of paper.*

**Reminder** Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to the instructor if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

**Problems** Please practice on exercises and solved problems in Chapter 5. The material they cover may appear on exams.

1. (**Sorting TM**) A TM *correctly sorts its input* if, given a comma-separated list of binary numbers, it halts with a sorted version of that list on its tape. (It does not matter what it does on other inputs.) Consider the problem of determining whether a TM correctly sorts its input. Formulate this problem as a language and prove it is undecidable.
2. (**Enthusiastic TM**) Consider the problem of determining whether a given TM ever<sup>1</sup> writes "464" on three adjacent squares of its tape.
  - (a) Formulate this problem as a language  $ENTHUSIASTIC_{TM}$ .
  - (b) Show  $ENTHUSIASTIC_{TM}$  is undecidable.
  - (c) Prove that  $ENTHUSIASTIC_{TM}$  is Turing-recognizable.
  - (d) Is  $\overline{ENTHUSIASTIC_{TM}}$  Turing-recognizable? Prove or disprove.
3. (**CFL<sub>TM</sub>**) Let  $CFL_{TM} = \{\langle M \rangle \mid M \text{ is a TM that recognizes a context-free language}\}$ . Prove the following statements about  $CFL_{TM}$ .
  - (a)  $CFL_{TM}$  is not Turing-recognizable.
  - (b)  $\overline{CFL_{TM}}$  is not Turing-recognizable (i.e.,  $CFL_{TM}$  is not co-Turing-recognizable.)
- 4\*. (**Optional, no collaboration**) Book, 5.16.

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<sup>1</sup>on some input