

Homework 6 – Due Thursday, October 15, 2009 before the lecture

Please refer to the general information handout for the full homework policy and options. This homework contains 3 problems, worth 10 points each. *Your solution to each problem should be handed in on a separate sheet of paper.*

Reminder Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to the instructor if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Problems Please practice on exercises and solved problems in Chapter 4. The material they cover may appear on exams.

1. (Decidable languages)

- (a) Book, 4.19. *Hint:* Run a decider for a related language, described in class.
- (b) Book, 4.25. *Hint:* Similar to the exercise we did on Tuesday.

2. (Countable and uncountable sets and diagonalization)

- (a) Let \mathcal{F} be the set of all finite languages over alphabet $\{0, 1\}$. Show that \mathcal{F} is countable.
- (b) Let \mathcal{B} be the set of all infinite sequences over $\{0, 1\}$. Show that \mathcal{B} is uncountable, using a proof by diagonalization.
- (c) Your friend told you that he found a new C++ library that contains many useful functions. One example is a function `halt` which takes two arguments: a program `main.cpp` and a valid input string x for that program. It returns 1 if `main.cpp` produces an output on input string x , and returns 0 if `main.cpp` runs forever on x . You want to convince your friend that `halt` cannot be always correct. However, your friend does not want to hear about TMs because they cannot possibly be relevant to C++ programs.

Give a diagonalization argument (similar to that on p. 179 of Sipser) to convince your friend that function `halt`, as specified above, does not exist. Your analogue of TM D on p. 179 should be a C++ program `src.cpp`. *Caution:* You cannot model it directly on TM D because your friend's claim is analogous to saying that HALT_{TM} is decidable, not that A_{TM} is decidable. Your program may call function `fread` that reads a file and returns the content of a file as a string. If you do not know C++, you can use any programming language with C-like syntax.

3. (Undecidable languages)

- (a) Book, 5.13.
- (b) Let $L = \{\langle M \rangle \mid M \text{ is a TM that accepts some string with more 1s than 0s}\}$. Show that L is undecidable.