

Homework 4 – Due Thursday, September 24, 2009 before the lecture

Please refer to the general information handout for the full homework policy and options. This homework contains 3 problems, worth 10 points each. *Your solution to each problem should be handed in on a separate sheet of paper.*

Reminder Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to the instructor if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Problems Please practice on exercises and solved problems in Chapter 2. The material they cover may appear on exams.

1. (**CFGs**) In the following parts, to prove that a grammar generates a particular language L you have to show that: (1) each string in L is generated and (2) each generated string is in L .

(a) Prove that the following grammar generates the complement of $\{0^n 1^n \mid n \geq 0\}$:

$$\begin{aligned} S &\rightarrow 0S1 \mid 1Y \mid Y0 \\ Y &\rightarrow 0Y \mid 1Y \mid \epsilon \end{aligned}$$

(b) Give a context-free grammar that generates $\{w \mid w \in \{a,b\}^* \text{ and in every prefix of } w \text{ the number of } a\text{'s is at least the number of } b\text{'s}\}$. Prove that your grammar is correct.

2. (**PDA**s)

(a) Book, 2.23. (I can be convinced to give a hint on Tuesday during the lecture if you explain where you are stuck.)

(b) Book, 2.44.

3. (**Non-CFL**s) Prove that the following languages are not context-free.

(a) Book, 2.30d.

(b) $\{0^k \mid k \text{ is a prime number}\}$.

(c) Book, 2.32.