
Homework 3 – Due Thursday, September 17, 2009 before the lecture

Please refer to the general information handout for the full homework policy and options.

Your solution to each problem should be handed in on a separate sheet of paper.

Reminder Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to the instructor if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Problems Please practice on exercises and solved problems in Chapter 2. The material they cover may appear on exams.

1. **(CFGs, 10 points, 1 page limit)** Give context-free grammars that generate the following languages. In parts (a) and (b), the alphabet Σ is $\{a, b\}$.
 - (a) $\{w \mid w = w^R, \text{ that is, } w \text{ is a palindrome}\}$. Recall that w^R represents string w written backwards. Palindromes can be of odd or even length.
 - (b) $\{w \mid \text{the number of } a\text{'s and } b\text{'s in } w \text{ are equal}\}$.
 - (c) $\{x_1 \# x_2 \# \dots \# x_k \mid k \geq 1, \text{ each } x_i \in \{a, b\}^*, \text{ and for some } i \text{ and } j, x_i = x_j^R\}$. The alphabet $\Sigma = \{a, b, \#\}$.
 - (d) $L_3 = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$.
 - (e) Is your grammar for L_3 ambiguous? Explain.
2. **(Closure properties, 15 points, 2 page limit)**
 - (a) Prove that the class of context-free languages is closed under the regular operations: union, concatenation and star.
 - (b) **(Erroneous proof of closure under $*$)** Give a counterexample to show that the following construction fails to prove that the class of context-free languages is closed under star. Let A be a CFL that is generated by the CFG $G = (V, \Sigma, R, S)$. Add the new rule $S \rightarrow SS \mid \epsilon$ and call the resulting grammar G' . This grammar is supposed to generate A^* .
 - (c^U) **(For undergraduate students¹: non-closure under \cap and complement)** Book, 2.2.
 - (c^G) **(For graduate students: non-closure under NOPREFIX and NOEXTEND)** Book, 2.41.
3. **(PDAs, 10 points, 1 page limit)**
 - (a) Give an informal description and the state diagram of a PDA recognizing the COMPLEMENT of the language $\{a^n b^n \mid n \geq 0\}$.
 - (b) Book, 2.25
- 4* **(Optional, no collaboration is allowed, 10 points, 1 page limit)** Book, 2.22.

¹Undergraduate students can solve either this question or the harder question below, but graduate students have to solve the harder question. Please hand in only one of the two.