CMPSC 464: *Intro to Theory of Computation* — Professor Sofya Raskhodnikova
Pennsylvania State University — August 25, 2009

Homework 0 – Due Thursday, August 27, 2008 before the lecture

Please refer to the general information handout for the full homework policy and options.

Page limit You can submit **at most** 1 page per problem, even if the problem has multiple parts. If you submit a longer solution for some problem, only the first page will be graded.

Reminder Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to the instructor if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

Exercises Please practice on exercises and solved problems in Chapters 0 and also on problem 0.11. The material they cover may appear on exams.

Problems

0. (**Bonus, 5 points**) Post a recognizable photo of your face on Angel. (Points will be awarded only for photos satisfying the requirements above.)

1^U (**For undergraduate students[1], proof techniques review, 10 points**) Suppose we are trying to divide a class of n students into groups of either 4 or 5 students.

(a) Find an error in the following proof that a class with $n \geq 8$ students can be divided into groups of 4 or 5. That is, identify the first incorrect sentence and explain what went wrong.

Proof. The proof is by strong induction. Let $P(n)$ be the proposition that a class with n students can be divided into teams of 4 or 5.

Base case: We prove that $P(n)$ is true for $n = 8, 9,$ and 10 by showing how to break classes of these sizes into groups of 4 or 5 students:

$$\begin{aligned} 8 &= 4+4; \\ 9 &= 4+5; \\ 10 &= 5+5. \end{aligned}$$

Induction hypothesis: Next, we must show that $P(8), ..., P(n)$ imply $P(n+1)$ for all $n > 10$. That is, we assume that $P(8), ..., P(n)$ are all true and show how to divide up a class of $n+1$ students into groups of 4 or 5. We first form one group of 4 students. Then we can divide the remaining $n-3$ students into groups of 4 or 5 by the assumption $P(n-3)$. This proves $P(n+1)$, and so the claim holds by induction. □

(b) Provide a correct strong induction proof that a class with $n \geq 12$ students can be divided into groups of 4 or 5.

1^G (**For graduate students, proof techniques review, 10 points**) Prove that in every tree with n vertices, there is a vertex v, such that if v and all edges adjacent to it are removed from the tree, it separates into connected components, each of size at most $2n/3$.

[1] Undergraduate students can solve either this question or the harder question below, but graduate students have to solve the harder question. Please hand in only one of the two.