CSE 586, Spring 2010
Advanced Computer Vision

Procrustes Shape Analysis
Credits

lots of slides are due to

Lecture material from Tim Cootes University of Manchester.
For more info, see http://www.isbe.man.ac.uk/~bim/
(includes code for exploring active shape/appearance models).
Overview

Statistical Shape Models: a method of modelling shape and shape variation

Active Shape Models: an active contour method with constraints on shape
Shape

- Need to model the variability in shape
- What is shape?
  - Geometric information that remains when location, scale and rotational effects removed (Kendall)

Same Shape  Different Shape
Shape

• More generally
  – *Shape is the geometric information invariant to a particular class of transformations*

• Transformations:
  – Euclidean (translation + rotation)
  – Similarity (translation+rotation+scaling)
  – Affine
  – Projective
# Shape

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<th>Shapes</th>
<th>Euclidean</th>
<th>Similarity</th>
<th>Affine</th>
<th>Projective</th>
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Shape Models

• We will represent the shape using a set of points
• We will model the variation by computing the PDF of the distribution of shapes in a training set (Gaussian)
• This allows us to generate new shapes similar to the training set
Point-based Shape Models

• Require labeled training images
  – point “landmarks” represent correspondences

  e.g. point 23 is always right corner of mouth
Suitable Landmarks

• Define correspondences
  – Well defined corners
  – ‘T’ junctions
  – Easily located biological landmarks
  – Use additional points along boundaries to define shape more accurately
Building Shape Models

- For each example

\[ \mathbf{x} = (x_1, y_1, \ldots, x_n, y_n)^T \]

Note, we are considering only 2D points here, but the approach generalizes to nD.
Statistical Shape Models

- Given a set of shapes:
  - Align shapes into common frame
    - Procrustes analysis
  - Estimate shape distribution $p(x)$
    - Single gaussian often sufficient
    - Mixture models sometimes necessary
Procrustes Analysis

Align one shape with another (not symmetric)

General Procrustes Analysis

Align a set of shapes with respect to some unknown “mean” shape (independent of ordering of shapes)
Why “Procrustes”? 

http://www.procrustes.nl/gif/illustr.gif
Aligning Two Shapes

• Procrustes analysis:
  – Find transformation which minimises

\[ |\mathbf{x}_1 - T(\mathbf{x}_2)|^2 \]

  – T is a particular type of transformation that you want “shape” to be invariant to.
Aligning Two Shapes

• Procrustes analysis:
  – Find transformation which minimises

  \[ | \mathbf{x}_1 - T(\mathbf{x}_2) |^2 \]

  – If T is a similarity transformation, the resulting shapes have
    • Identical center of mass
    • approximately the same scale and orientation
Steps in Similarity Alignment

Given a set of K points:  **Configuration**

Translation normalization:  **Centered Configuration**
   (center of mass at origin)

Scale normalization:  **Pre-shape**
   (divide by Sqrt of SSQ centered coordinates)

Rotation normalization:  **Shape**
   (rotate to alignment with ref shape)
Aligning Pre-Shapes by Rotation

Sketch:

A, B are two Kx2 preshapes, R is unknown rotation

Want to minimize $\|(AR - B)^2\|$ subject to $R^TR=I$

After some manipulation, we get


Note, both sides are symmetric and have same eigenvalues. This is a job for SVD!
Aligning Pre-Shapes by Rotation

Sketch continued:

\[(A^T B)(A^T B)^T = R (A^T B)^T (A^T B) R^T\]

\[
\begin{align*}
V \quad D \quad V^T &= R \quad W \quad D \quad W^T \quad R^T \\
\end{align*}
\]

So... \( V = R \quad W \)

and therefore \( R = V \quad W^T \)
Recall:

Procrustes Analysis

Align one shape with another (not symmetric)

General Procrustes Analysis

Align a set of shapes with respect to some unknown “mean” shape (independent of ordering of shapes)
Aligning a Set of Shapes

- Generalised Procrustes Analysis
  - Find the transformations \( T_i \) which minimise

\[
\sum |m - T_i(x_i)|^2
\]

- Where \( m = \frac{1}{n} \sum T_i(x_i) \)

- Under the constraint that \( |m| = 1 \)
Aligning Shapes: Algorithm

- Normalise all so center of mass is at origin, and size=1
- Let \( m = x_1 \)
- Align each shape with \( m \) (via a rotation)
- Re-calculate \( m = \frac{1}{n} \sum T_i(x_i) \)
- Normalise \( m \) to default size, orientation
- Repeat until convergence