

CSE586 Assignment 3, due March 17 Thurs

- 1) Prove that the set of all 2x2 matrices of the form $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$ form a group under matrix multiplication.
- 2) Prove that the set of all complex numbers of the form $e^{i\theta}$ form a group under scalar multiplication. You should NOT use any $\sin(\theta)$ or $\cos(\theta)$ terms in this proof.
3. After two 2D landmark point sets $P=\{(x_i, y_i), i=1\dots n\}$ and $Q=\{(x'_i, y'_i), i=1\dots n\}$ have been "centered" by subtracting off their respective centers of mass, solving for the similarity transformation between P and Q reduces to solving for a scaled rotation transformation, since the translation between them is now known to be 0). Writing a scaled rotation as a 2x2 matrix $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ where $a = s\cos(\theta)$ and $b = s\sin(\theta)$, set up and solve a least squares system of equations to find a and b as a function of the x_i, y_i, x'_i and y'_i coordinates.
4. Write some code using any of the methods we talked about in class to find the similarity transformation (translation, rotation and scale) between two 2D landmark point sets $P=\{(x_i, y_i), i=1\dots n\}$ and $Q=\{(x'_i, y'_i), i=1\dots n\}$.
5. Use the code from part 4 as a subroutine for doing generalize procrustes analysis. That is, given a set of K 2D landmark point sets P_1, P_2, \dots, P_K , find their "mean shape" M and the set of K similarity transformations that best aligns them with that "mean shape". I have put a dataset called "schizodata.mat" on our website, which you can use as test data. This data is due to Fred Bookstein, who was using it to try to find a diagnostic tool to distinguish schizophrenic from normal brains, as described in the article "The Shape of Madness" (also on the website). You could try it on any other data that is of interest to you, of course.
6. There will be a part 6, which will involve using PCA to find the primary modes of shape deformation using output from the program in part 5. I'll flesh this out more next week after we have had the relevant lectures.