Lecture 18:
Generalized Stereo:
Epipolar Geometry

Key idea: Any two images showing an overlapping view of the world can be treated as a stereo pair...

... we just have to figure out how the two views are related.

Some of the most “beautiful” math in vision concerns describing how multiple views are related, geometrically.

Recall: Epipolar Constraint

Important Stereo Vision Concept:

Given a point in the left image, we don’t have to search the whole right image for a corresponding point.

The “epipolar constraint” reduces the search space to a one-dimensional line.

Review: Simple Stereo System

Same Y Coord!

Equation relating depth and disparity

General Stereo

In general, the cameras may be related by an arbitrary transformation (R,T)

Epipolar Matrix

In general, intrinsic camera parameters may be different, and even unknown

Fundamental Matrix

Recall: Epipolar Constraint

Equation relating depth and disparity

Review: Epipolar Constraint

Corresponding features are constrained to lie along conjugate epipolar lines (on the same row in the case of our simple setup).
EPIPOLAR GEOMETRY

Epipolar Geometry

Epipoles:
• \(e_L\): left image of \(O_L\)
• \(e_R\): right image of \(O_R\)

Epipolar plane:
• Three points: \(O_L, O_R,\) and \(P\) define an epipolar plane

Epipolar lines and epipolar constraint:
• Intersections of epipolar plane with the image planes
• Corresponding points are on "conjugate" epipolar lines

EPIPOLAR GEOMETRY

BORING!!!

Let's try again...

Epipolar Geometry

A Visualization

Would would Pinhead’s eye look like close up?

answer

Epipolar Geometry

Rays to Points in Scene

Tie threads on to the pins and connect focal point to scene points

Now what would this look like to a second observer?
Rays Seen from Second Observer

Rays Seen by the First Viewer

Epipolar Geometry

Epipole not Necessarily in Image

Epipole: location of cam2 as seen by cam1.

Epipole: location of cam1 as seen by cam2.

Corresponding points lie on conjugate epipolar lines.

Conjugate epipolar lines induce a generalized 1D “scan-line” ordering on the images (analogous to traditional scan line ordering of rows in an image).
Epipolar Geometry

Epipoles:
• $e_l$: left image of $O_l$
• $e_r$: right image of $O_r$

Epipolar plane:
• Three points: $O_l, O_r,$ and $P$ define an epipolar plane
• Intersections of epipolar plane with the image planes
• Corresponding points are on "conjugate" epipolar lines

Epipolar Constraint:
Given Epipoles:
• $e_l$: left image of $O_l$
• $e_r$: right image of $O_r$

Given $p$:
• Consider its epipolar line: $p_l e_l$
• Find epipolar plane: $O_l p_l e_l$
• Intersect the epipolar plane with the right image plane
• Search for $p_r$ on the right epipolar line

Essential Matrix

$$P_r = R(P_l - T)$$

Does this look familiar? Recall world to camera transformation by $(R, T)$. Here, we are transforming from camera to camera.

Essential Matrix

$$P_l - T = R^{-1} P_r = R^T P_r$$

Essential Matrix

Epipolar constraint: $P_l$, $T$ and $P_r$ - $T$ are coplanar:

$$(P_l - T)^T \cdot T \times P_l = 0$$

$$P_l - T = R^T P_r \Rightarrow (R^T P_r)^T \cdot T \times P_l = 0$$

Essential Matrix

Epipolar constraint: $P_r$, $T$ and $P_l$ - $T$ are coplanar:

$$(R^T P_r)^T \cdot T \times P_l = 0$$

$$(P_r^T R) \cdot (T \times P_l) = 0$$
Vector Product as a Matrix Multiplication

\[ T \times P_l = \begin{vmatrix} i & j & k \\ T_x & T_y & T_z \\ P_{x_l} & P_{y_l} & P_{z_l} \end{vmatrix} \]

\[ T \times P_l = (T_y P_{x_l} - T_z P_{y_l})i + (T_z P_{x_l} - T_x P_{z_l})j + (T_x P_{y_l} - T_y P_{z_l})k \]

\[ T \times P_l = S P_l = \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{bmatrix} \begin{bmatrix} P_{x_l} \\ P_{y_l} \\ P_{z_l} \end{bmatrix} = \begin{bmatrix} T_y P_{x_l} - T_z P_{y_l} \\ T_z P_{x_l} - T_x P_{z_l} \\ T_x P_{y_l} - T_y P_{z_l} \end{bmatrix} \]

\( S \) has rank 2; it depends only on \( T \)

Essential Matrix

Epipolar constraint: \( P_l, T \) and \( P_l - T \) are coplanar:

\[ (P^T_T \overrightarrow{R}) \cdot (T \times P_l) = 0 \]

\[ P^T_T R S P_l = 0 \]

Essential Matrix Properties

\[ E = RS \]

- has rank 2
- depends only on the EXTRINSIC Parameters (\( R \) & \( T \))

We will discuss more of the wonderful properties of this matrix next time...