Lecture 18:
Generalized Stereo:
Epipolar Geometry

Key idea: Any two images showing an overlapping view of the world can be treated as a stereo pair...
... we just have to figure out how the two views are related.

Some of the most “beautiful” math in vision concerns describing how multiple views are related, geometrically.

Recall: Epipolar Constraint
Important Stereo Vision Concept:
Given a point in the left image, we don’t have to search the whole right image for a corresponding point.
The “epipolar constraint” reduces the search space to a one-dimensional line.

Review: Simple Stereo System

In general, the cameras may be related by an arbitrary transformation (R, T)

Epipolar Matrix
In general, intrinsic camera parameters may be different, and even unknown

Fundamental Matrix
EPIPOLAR GEOMETRY

Epipolar Geometry

Epipoles:
• \( e_l \): left image of \( O_l \)
• \( e_r \): right image of \( O_r \)

Epipolar plane:
• Three points: \( O_l, O_r, \) and \( P \) define an epipolar plane

Epipolar lines and epipolar constraint:
• Intersections of epipolar plane with the image planes
• Corresponding points are on "conjugate" epipolar lines

Boring!!!

Let's try again...

Epipolar Geometry

Epipolar Geometry

A Visualization

Would would Pinhead’s eye look like close up?

Rays to Points in Scene

Now what would this look like to a second observer?
Rays Seen from Second Observer

Rays Seen by the First Viewer

Epipolar Geometry

Epipole not Necessarily in Image

Epipole : location of cam2 as seen by cam1.

Epipole : location of cam1 as seen by cam2.

Corresponding points lie on conjugate epipolar lines

Conjugate epipolar lines induce a generalized 1D “scan-line” ordering on the images (analogous to traditional scan line ordering of rows in an image)
Epipolar Geometry

Epipoles:
- $e_l$: left image of $O_l$
- $e_r$: right image of $O_r$

Epipolar plane:
- Three points: $O_l, O_r, P$ define an epipolar plane
- Intersections of epipolar plane with the image planes
- Corresponding points are on "conjugate" epipolar lines

Epipolar Constraint:
- Intersections of epipolar plane with the right image plane
- Corresponding points are on the right epipolar line

Essential Matrix

Given epipoles:
- $e_l$: left image of $O_l$
- $e_r$: right image of $O_r$

Given $p_l$:
- Consider its epipolar line: $p_l, e_l$
- Find epipolar plane: $O_l, p_l, e_l$
- Intersect the epipolar plane with the right image plane
- Search for $p_r$ on the right epipolar line

Essential Matrix

$P_r = R(P_l - T)$

Does this look familiar? Recall world to camera transformation by $(R,T)$. Here, we are transforming from camera to camera.

Essential Matrix

Epipolar constraint: $P_l, T$ and $P_r, T$ are coplanar:

$P_l - T = R^TP_r$; $R^TP_r = R^TP_r$

Epipolar constraint: $P_l, T$ and $P_r, T$ are coplanar:

$(P_l - T)^T \cdot T \times P_l = 0$

$(R^TP_r)^T \cdot T \times P_l = 0$

$(P_r^TR) \cdot (T \times P_l) = 0$
Vector Product as a Matrix Multiplication

\[
T \times P_i = \begin{bmatrix}
i & j & k \\
T_x & T_y & T_z \\
P_{x_i} & P_{y_i} & P_{z_i}
\end{bmatrix}
\]

\[
T \times P_i = (T_x P_{x_i} - T_z P_{y_i})\mathbf{i} + (T_y P_{x_i} - T_x P_{y_i})\mathbf{j} + (T_z P_{x_i} - T_y P_{y_i})\mathbf{k}
\]

\[
T \times P_i = SP_i = \begin{bmatrix}
0 & -T_z & T_y \\
T_z & 0 & -T_x \\
-T_y & T_x & 0
\end{bmatrix}
\begin{bmatrix}
P_{x_i} \\
P_{y_i} \\
P_{z_i}
\end{bmatrix}
\]

\[
S \text{ has rank 2; it depends only on } T
\]

Essential Matrix

Epipolar constraint: \(P_i, T\) and \(P_i - T\) are coplanar:

\[
(P^T R) \cdot (T \times P_i) = 0
\]

\[
P^T R S P_i = 0
\]

Essential Matrix Properties

\[E = RS\]

- has rank 2
- depends only on the EXTRINSIC Parameters (R & T)

We will discuss more of the wonderful properties of this matrix next time...