**Problem 1:** Let \( L \) be the set of words over the alphabet \( \{a, b, c\} \) that have equal number of \( a \)'s, \( b \)'s and \( c \)'s — if counted modulo 7. Define a DFA \( A \) such that \( L = L(A) \).

**Problem 2:** Assume that \( \Sigma = \{0, 1, \cdots, 9\} \). Define function \( \alpha \) recursively: \( \alpha(\lambda) = \emptyset \), \( \alpha(ua) = \alpha(u) \cup \{a\} \). For a set \( S \subseteq \Sigma \) define \( L_S = \{u \in \Sigma^* : \alpha(u) = S\} \).

\( a. \) What is the set of quotients of the language \( L_\Sigma \)?

\( b. \) Define succinctly a minimal DFA for \( L_\Sigma \).

\( c. \) Define succinctly a minimal DFA for \( \Sigma^* - L_\Sigma \).

\( d. \) Define an NFA for \( \Sigma^* - L_\Sigma \) with 11 states.

**Problem 3:** \( L \subseteq \{0, 1\} \) is a set of reversed binary codes of numbers that are equal to 1 modulo 7. The following DFA accepts \( L \):

\[
Q = \{0,1,2,3,4,5,6\} \times \{0,1,2,3,4,5,6\} \\
q_0 = (0,1) \\
\delta((r, p), b) = (r + bp \mod 7, 2p \mod 7) \\
A = \{1\} \times \{0,1,2,3,4,5,6\}
\]

The idea of this automaton is the following: \( \hat{\delta}(q_0, u) = (\text{val}(u), 2^{|u|}) \), where \( \text{val}(u) \) is the binary value of \( u^R \). For example, \( \hat{\delta}(q_0, 01101) = (22 \mod 7, 32 \mod 7) = (1, 4) \), and because 22 equals 1 modulo 7 it should be accepted. In other words, after reading \( u \) we compute the value of its reverse modulo 7 and the power of 2 that should be used to compute the next value. If the next bit is zero, it is a zero on the most significant position so the value is unchanged, and if the next bit is one, we have to add the value of the current power of 2, modulo 7.

\( a. \) Find the set of reachable states of this automaton.

\( b. \) Find the partition of the reachable states into equivalent classes.

\( c. \) Define a minimal DFA that is equivalent to this automaton.

**Problem 4:** Let \( \text{dist}(u, v) \) equal \( \infty \) if \( |u| \neq |v| \) and otherwise let it be the Hamming distance of \( u \) and \( v \). Let

\[
\text{dist}(u, L) = \min_{v \in L} \text{dist}(u, v) \\
\text{Dist}(L, k) = \{u \in \Sigma^* : \text{dist}(u, L) = k\}
\]

Show that if \( L \) is regular, \( \text{Dist}(L, 3) \) is also regular. Try to use closure properties of DFA rather than a conditional construction of a finite automaton.