Solve exercises from the textbook:

On page 229, 1 (heap construction the maximum at the root)

Answer.

a.

```
        8
      /   \
     6     8
    /\   /\  \
   3  1  5  7
```

c. No, we can try another sequence, 2, 3, 4, 6, 5, 1, 0.

On page 229, 2 (heap checking)

Answer.

```java
for (i = 2; i <= n; i++)
    if (H[i] > H[i/2])
        return 0;
return 1;
```

The running time is $O(n)$. 
On page 246, 10 (post office location, use problem of computing median),

**Answer.** The first observation is that under some circumstances we can easily improve the location of the post-office. If the post office is at \((x, y)\) and there are \(a > n/2\) input points with \(x_i < x\), we can change \(x\) into \(x - \varepsilon\), and in the summation \(\sum_{i=1}^{n} |x_i - x|\) we will have \(a\) terms decreased by \(\varepsilon\) and at most \(n - a < a\) terms increased by \(\varepsilon\), thus the summation will decrease.

When such easy (if small) improvement is not possible? When the number of \(x_i\)’s smaller than \(x\) is at most \(n/2\), and the number of \(x_i\)’s larger than \(x\) is at most \(n/2\). Thus \(x\) is the **median** of \(x_i\)’s.

Similarly, \(y\) has to be a median of \(y_i\)’s.

In other words, we reduced the problem of best location for the post-office to the median problem (and we can solve it in time \(O(n)\)).

On page 276, 3.

**Answer.** Order \(m\) of the B-tree is the maximum branching of a node (number of children), and \(m/2\) is the minimum branching of a non-root. The minimum non-root subtree with one level has at least \(m/2\) keys, the minimum non-root subtree with two levels has at least \((m/2)^2\) keys, and in general, the minimum non-root tree with \(L\) levels has at least \((m/2)^L\) keys. The minimum tree with \(L\) levels has two subtrees, each with \(L - 1\) levels and thus altogether at least \(2 \times (m/2)^{L-1}\) keys.

To guarantee at most \(L\) levels we should have at most \(2 \times (m/2)^L\) keys.

We want to guarantee at most 4 levels (3 on the disk), thus \(2 \times (m/2)^4 \geq 10^8\) \(m \geq 10 \times 2^{0.75} \approx 168\).
On page 276, 4.

Answer.