Homework 2. Programming Language Design & Analysis (CSE 497)

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Due date: Sept. 8th in class.

1. (2 points). In class, we gave two different presentations of terms of the simple arithmetic language: first as the smallest set that is closed under certain rules \((T)\), and second as the limit of a series of sets \((S)\). Prove these two presentations are equivalent.

2. (5 points). The following language is the simple boolean language we have discussed in class:

\[
\begin{align*}
  t & ::= \text{true} \mid \text{false} \mid \text{if } t \text{ then } t \text{ else } t \\
  v & ::= \text{true} \mid \text{false}
\end{align*}
\]

We next define a set of evaluation rules for the language:

\[
\begin{align*}
  \text{if true then } v_2 \text{ else } v_3 & \rightarrow v_2 \\
  \text{if false then } v_2 \text{ else } v_3 & \rightarrow v_3 \\
  t_2 & \rightarrow t'_2 \\
  \text{if } t_1 \text{ then } t_2 \text{ else } t_3 & \rightarrow \text{if } t_1 \text{ then } t'_2 \text{ else } t_3 \\
  t_3 & \rightarrow t'_3 \\
  \text{if } t_1 \text{ then } v_2 \text{ else } t_3 & \rightarrow \text{if } t_1 \text{ then } v_2 \text{ else } t'_3 \\
  t_1 & \rightarrow t'_1 \\
  \text{if } t_1 \text{ then } v_2 \text{ else } v_3 & \rightarrow \text{if } t'_1 \text{ then } v_2 \text{ else } v_3
\end{align*}
\]

This evaluation strategy is different from the one in Fig.3-1 of the textbook in that the then and else branches of an if get evaluated (in that order) before the guard.

(a) Write down the evaluation steps for the following term according to the new
system. Write every step until you get a normal form.

\[
\text{if } (\text{if true then true else false}) \\
\quad \text{then } (\text{if false then true else false}) \\
\quad \text{else } (\text{if false then true else false})
\]

(b) Prove the theorem that if \( t \) is in normal form, then \( t \) is a value.

(c) Suppose that the evaluation of the else branch should come before the then branch. How should we adjust the rules and how would the term in (a) get evaluated in the new set of rules?

(d) Suppose that if the evaluation of the then and the else branches leads to the same value, we want to immediately produce that value (“short-circuiting” the evaluation of the guard). How should we change the rules? How would the term in (a) get evaluated in the new set of rules?