

Computer Science/Mathematics 550
Numerical Algebra
Assignment Four
Due 31 October 2007

You are going to write a MATLAB function for performing preconditioned conjugate gradient on the symmetric positive definite Toeplitz system

$$T_n \mathbf{x} = \mathbf{b}.$$

You will test it with

- No preconditioning
- Strang's preconditioner
- T. Chan's preconditioner

You should use the “four FFT” approach on p.49 of your text to implement the preconditioner. It is up to you whether to do this with one m-file or three (obviously, you may use more). A symmetric $n \times n$ Toeplitz matrix should be represented as a vector $\mathbf{t} = (t_0, \dots, t_{n-1})^T$ (although it may be more convenient in MATLAB to index the diagonals as t_1, \dots, t_n). You should **never** have to store the actual matrix.

The test problem will be a 1000×1000 symmetric banded Toeplitz matrix with only t_0, \dots, t_{99} nonzero. It should be generated from the three MATLAB commands

```
that=randn(99,1); %  $t_1, \dots, t_{99}$  random
t0=sum(abs(that))+1e-4; % This will make  $T$  diagonally dominant and positive definite
t=[t0;that]; % This vector represents the Toeplitz system
```

All matrix vector multiplications with T should be done with FFTs! Become familiar with the MATLAB functions **fft** and **ifft**. Set up the right hand side \mathbf{b} such that

$$T\mathbf{x} = \mathbf{b}$$

and the exact solution $\mathbf{x} = (1, 1, \dots, 1)^T$ (from MATLAB function **ones**). Use the starting vector $\mathbf{x}_0 = (0, 0, \dots, 0)^T$ (from MATLAB function **ones**). Iterate until the residual $\mathbf{r} = \mathbf{b} - T\mathbf{x}$ satisfies $\|\mathbf{r}\|_2 \leq 10^{-3}$ or until 200 iterations is reached. Try to update this residual with as few calls to the FFT routine as possible!

Please turn in results on number of iterations and error and your code. I do not need printouts of all of the numbers generated by algorithm.