CSE451.1   HOMEWORK3 SOLUTION (10+10+10+10=40)

1.

$$4x_1 + x_2 = 4.1$$
$$x_1 + 4x_2 + x_3 = 2.4$$
$$x_2 + 4x_3 + x_4 = 4.2$$
$$x_3 + 4x_4 + x_5 = 2.4$$
$$x_4 + 4x_5 = 4.1$$

Perform Jacobi Iteration:

$$x_1 = \frac{4.1 - x_2}{4}$$
$$x_2 = \frac{2.4 - x_1 - x_3}{4}$$
$$x_3 = \frac{4.2 - x_2 - x_4}{4}$$
$$x_4 = \frac{2.4 - x_3 - x_5}{4}$$
$$x_5 = \frac{4.1 - x_4}{4}$$

Initial guess $\vec{x} = 0$,

$$x_1 = 1.025$$
$$x_2 = 0.6$$
$$x_3 = 1.05$$
$$x_4 = 0.6$$
$$x_5 = 1.025$$

2nd Iteration

$$x_1 = 0.8750$$
$$x_2 = 0.08125$$
$$x_3 = 0.7500$$
$$x_4 = 0.08125$$
$$x_5 = 0.8750$$

3rd Iteration

$x_1 = 1.0047$
$x_2 = 0.1938$
$x_3 = 1.0094$
$x_4 = 0.1938$
$x_5 = 1.0047$

2.
We have same initial guess as before.
1st iteration

$$x_1 = \frac{4.1-0}{4} = 1.025$$
$$x_2 = \frac{2.4-1.025-0}{4} = 0.34375$$
$$x_3 = \frac{4.2-0.34375-0}{4} = 0.964063$$
$$x_4 = \frac{2.4-0.964063-0}{4} = 0.358984$$
$$x_5 = \frac{4.1-0.358984}{4} = 0.935254$$

2nd iteration

$$x_1 = \frac{4.1-0.34375}{4} = 0.939063$$
$$x_2 = \frac{2.4-0.939063-0.964063}{4} = 0.124219$$
$$x_3 = \frac{4.2-0.124219-0.358984}{4} = 0.929199$$
$$x_4 = \frac{2.4-0.929199-0.935254}{4} = 0.133887$$
$$x_5 = \frac{4.1-0.133887}{4} = 0.991528$$

3rd iteration

$$x_1 = \frac{4.1 - 0.124219}{4} = 0.993945$$
$$x_2 = \frac{2.4 - 0.993945 - 0.929199}{4} = 0.119214$$
$$x_3 = \frac{4.2 - 0.119214 - 0.133887}{4} = 0.986725$$
$$x_4 = \frac{2.4 - 0.986725 - 0.991528}{4} = 0.105437$$
$$x_5 = \frac{4.1 - 0.105437}{4} = 0.998641$$

Gauss-Seidel appears better than Jacobi because it converges faster and gives more accurate answer after three iterations.

3.

a) For the matrix A,

We use *max(sum(abs(A)))* to find the column(s) with maximum absolute sum: Column 2 and 3.

Hence every vector x=(a b c)' with a=0, b+c=1 satisfies the requirement.

eg. $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ -1/4 \\ 3/4 \end{pmatrix}, etc.$

Similarly, for Hilbert matrix H,

$$x = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

b)

For A^{-1}, $x = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

For $H^{-1}, x = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$.

c)

Matlab Code:

[infinorm, i]=max(sum(abs(A')));
x=sign(A(i,:))'

For A, $x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

For H, $x = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$.

d)

For A^{-1}, $x = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$,.

For $H^{-1}, x = \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$.

4.

$\| A \|_1 = 6, \| A^{-1} \| = 0.4808.$

Condition number = $\| A^{-1} \|_1 * \| A \|_1 = 2.8846.$