Computer Science/Mathematics 455
Assignment Five
Due 15 November 2004

1. How to compute $\pi$
   (a) Show that
   \[ P_0(h) = h^{-1} \sin \pi h = \pi + a_2 h^2 + a_4 h^4 + \ldots + a_{2n} h^{2n} + O(h^{2n+2}) \]
   for some coefficients $a_2, a_4, \ldots, a_{2n}, \ldots$.
   (b) Using Richardson Extrapolation (ala Romberg integration) show how to get an $O(h^{2n+2})$ approximation to $\pi$ from $P_0(h), P_0(h/2), \ldots, P_0(h/2^n)$.
   (c) We can use this algorithm for $\pi$ without computing the sin function. We know that
   \[ \sin \frac{\pi}{4} = \sqrt{0.5}, \]
   and from the formulas
   \[ \cos \theta = \sqrt{1 - \sin^2 \theta} \]
   \[ \sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} \]
   we can compute $\sin \pi h$, for $h = 1/4, 1/8, \ldots 1/2^k$ without using the sin function. Use this to compute $P_0(1/4), P_0(1/8), P_0(1/16), P_0(1/32)$ without using the sin function either in MATLAB or on your calculator (or any place else). Then use the extrapolation algorithm from the previous problem to improve your approximation to $\pi$. How much better is the extrapolated approximation than $P_0(1/32)$?

2. Problem 2, p.114
3. Problem 4, p.114
5. Problem 4, p.132 (except do not do midpoint rule).
6. Problem 2, p. 152
7. Problem 1, p.159