Computer Science/Mathematics 455
Assignment Five
Due 12 April 2004

1. How to compute $\pi$

(a) Show that

$$P_0(h) = h^{-1} \sin \pi h = \pi + a_2 h^2 + a_4 h^4 + \ldots + a_{2n} h^{2n} + O(h^{2n+2})$$

for some coefficients $a_2, a_4, \ldots, a_{2n}, \ldots$.

(b) Using Richardson Extrapolation (ala Romberg integration) show how to get an $O(h^{2n+2})$ approximation to $\pi$ from $P_0(h), P_0(h/2), \ldots, P_0(h/2^n)$.

(c) We can use this algorithm for $\pi$ without computing the sin function. We know that

$$\sin \frac{\pi}{4} = \sqrt{0.5},$$

and from the formulas

$$\cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}}$$

we can compute $\sin \pi h$, for $h = 1/4, 1/8, \ldots 1/2^k$ without using the sin function. Use this to compute $P_0(1/4), P_0(1/8), P_0(1/16), P_0(1/32)$ without using the sin function either in MATLAB or on your calculator (or any place else). Then use the extrapolation algorithm from the previous problem to improve your approximation to $\pi$. How much better is the extrapolated approximation than $P_0(1/32)$?

2. Problem 2, p.131.

3. Problem 4, p.132 (except do not do midpoint rule).

4. Problem 6, p.139 (again, do not do midpoint rule).

5. Problem 2, p. 152

6. Problem 1, p.159

7. Problem 4, p. 159.