
Homework 12 – Due Friday, December 12, 2008

Please refer to the general information handout for the full homework policy and options.

Reminders

- Your solutions are due before the lecture. Late homework will not be accepted.
- Collaboration is permitted, but you must write the solutions *by yourself without assistance*, and be ready to explain them orally to a member of the course staff if asked. You must also identify your collaborators. *Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.*
- To facilitate grading, please write down your solution to each problem on a separate sheet of paper. Make sure to include all identifying information and your collaborators on each sheet. Your solutions to different problems will be graded separately, possibly by different people, and returned to you independently of each other.
- For problems that require you to provide an algorithm, you must give a precise description of the algorithm, **together with a proof of correctness** and an analysis of its running time. You may use algorithms from class as subroutines. You may also use any facts that we proved in class or from the book.

Reminder To show that a problem is NP-complete, you must show *both* that it is in NP and that it is NP-hard. The easiest way to show that problem X is NP-hard is to prove $Y \leq_{p,Karp} X$ (“there is a Karp reduction from Y to X ” or “ Y Karp-reduces to X ”), where Y is an NP-hard problem already covered in class.

Problems to be handed in

<p>Page limits: The answer to each problem should fit in 2 pages (or one double-sided sheet) of paper. Longer answers will be penalized.</p>
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1. You are faced with a new decision problem, code-named X by the secretive government organization for which you are consulting. For each of the following statements, say whether it is (i) true, (ii) false, or (iii) unknown given the current state-of-the-art. For example, the statement $P = NP$ falls into category (iii).

Justify your answers briefly (one sentence).

- (a) If $X \leq_{p,Karp} SAT$, then $X \in NP$.
- (b) If $X \leq_{p,Cook} SAT$, then $X \in NP$.
- (c) If X Karp-reduces to SAT and X is in NP, then X is NP-complete.
- (d) If $SAT \leq_{p,Karp} X$ and X is in NP, then X is NP-complete.

(Continued on next page.)

- (e) If X is NP-complete, then X Karp-reduces to perfect bipartite matching.
 - (f) If X is NP-complete, then perfect bipartite matching Karp-reduces to X .
2. We saw in class that 3-SAT is NP-complete. Consider instead 2-SAT, in which the input is a formula with at most 2 literals per clause. Show that 2-SAT is in P. (*Hint*: Reduce to the problem of testing whether a given graph is bipartite, discussed in Section 3.4.)
 3. KT book, Chapter 8, problem 8 (Madison's letters). You can reduce from 3-dimensional matching, defined in Sec. 8.6 (although a reduction from any NP-complete problem discussed in the book will do).