Homework 1 – Due Friday, January 13, 2012

Reminders Collaboration is permitted, but you must write the solutions by yourself without assistance, and be ready to explain them orally to a member of the course staff if asked. You must also identify your collaborators. Getting solutions from outside sources such as the Web or students not enrolled in the class is strictly forbidden.

When you describe an algorithm, remember that clarity is the main goal. Unless otherwise specified, you must give pseudocode for your algorithm as well as a short English-language description.

Exercises These should not be handed in, but the material they cover may appear on exams.

- CLRS (textbook) chapter 1 exercises.
- Explain the error in the following proof.
  
  **Claim:** All frisbees are the same color.
  
  **Proof:** We prove this by induction on \( n \), the number of frisbees in the universe. Base case: if there is only one frisbee, there is only one color. Induction step: Assume as induction hypothesis that within any set of \( n \) frisbees, there is only one color. Now look at any set of \( n + 1 \) frisbees. Number them 1, 2, 3, ..., \( n, n + 1 \). Consider the sets \( \{1, 2, 3, ..., n\} \) and \( \{2, 3, 4, ..., n + 1\} \). Each is a set of only \( n \) frisbees, therefore within each there is only one color. But the two sets overlap, so there must be only one color among all \( n + 1 \) frisbees.

Problems to be handed in. **Please submit each problem on a separate sheet of paper.**

1. (Reasoning by induction) You have a rectangular candy bar marked into \( m \times n \) squares, and you wish to break up the bar into individual squares. At each step, you may pick up one piece and break it along any of its marked vertical or horizontal lines. Prove by induction that every method finishes in the same number of steps. *[Hint: Try to figure out what the right number of steps is (as a function of \( m \) and \( n \)) by picking a particular strategy. Then use strong induction to prove that every method uses that number of steps.]*

2. (Writing Algorithms) Consider the following problem: given an array of real numbers \( a_1, ..., a_n \), and a real number \( y \), evaluate the polynomial \( f(x) = \sum_{i=1}^{n} a_i x^i \) at the point \( y \) (that is, compute \( \sum_{i=1}^{n} a_i y^i \)).

   (a) Give an algorithm which solves this problem. Give a short (1 or 2 sentence) English-language description, as well as pseudocode for your algorithm. Explain why your algorithm is correct.

   (b) How many multiplications does your algorithm use, as a function of \( n \)?

   (Continued on next page.)
(c) Give an algorithm that solves the problem using at most $n$ multiplications (not $2n$ or even $n+1$). [Hint: Can you compute $a_1y + a_2y^2$ using only two multiplications?] Again, explain why your algorithm is correct.

Note: On most machines, addition is faster than multiplication, so optimizing the number of multiplications is important.