Modeling and Detecting Bidding Anomalies in Day-ahead Electricity Markets

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Abstract—Virtual bids were introduced in U.S. wholesale electricity markets to exploit arbitrage opportunities arising from expected price differences between day-ahead and real-time energy markets. These financial instruments have interactions with other elements of the electricity market design. For instance, virtual bids can affect day-ahead market-clearing prices so as to enhance the value of Financial Transmission Rights (FTRs) that settle at those energy prices. We consider a model of the day-ahead electricity market at one node in the network, under the assumption that real-time prices are not affected by virtual bidding. Theoretical results on interior Nash equilibria are given, assuming virtual bidders can perfectly predict real-time prices and hold no FTRs. We then adopt a hypergame framework to model the day-ahead market, assuming imperfect prediction of real-time prices by different virtual bidders, and present simulation results with and without FTRs. Finally, we discuss two detection mechanisms that could be used by regulators to distinguish between competitive and anti-competitive market outcomes, as well as trade-offs between specificity and sensitivity.

I. INTRODUCTION

About two thirds of electricity consumers in the United States are served by Regional Transmission Organizations (RTOs) and Independent System Operators (ISOs). One of their primary responsibilities is the operation of organized auctions for purchasing and selling electricity that have a two-settlement structure with coordinated day-ahead (DA) and real-time (RT) energy markets. The day-ahead market takes place on the day before the actual power dispatch, and creates a financial obligation to deliver and withdraw power from the transmission grid. In contrast, the real-time energy market is a physical market where predicted and actual supply and demand of electricity are balanced on the delivery day. In both auctions, the result is a market clearing with hourly locational marginal prices that reflect the short-run marginal cost of serving one incremental megawatt of load at each node on the transmission network [1]. Sales and purchases cleared at the day-ahead price that are not converted into physical positions must be bought or sold back at the real-time price.

Purely financial transactions, known as virtual (or convergence) bids, were introduced in wholesale electricity markets to allow participants (including energy traders that do not control generation assets or serve load) to exploit arbitrage opportunities arising from expected price differences between day-ahead and real-time energy markets, to help the convergence between DA and RT prices, and represent a common feature of wholesale electricity markets[2]. More specifically, virtual demand (supply) bids are financial positions for the purchase (sale) of energy in the day-ahead market, which are settled with a countervailing offer to sell (buy) at the real-time price, without the bidder taking title to physical electricity. Virtual demand bids are typically referred to as DECs, while virtual supply bids are known as INCs. Virtual bids clear with generation and load bids in the day-ahead market, and may settle the day-ahead locational marginal price.

Virtual bids have strong interactions with other elements of the electricity market design. For instance, Financial Transmission Rights (FTRs) are financial contracts to hedge transmission congestion between two nodes in the transmission network (a source and a sink defined in the contract), and entitle their holders the right to collect a payment when day-ahead congestion arises between the source and the sink [3]. Since FTRs settle at the day-ahead prices, virtual bids could be placed in the day-ahead energy market in order to affect day-ahead electricity prices so as to enhance the value of FTRs. The strategic use of virtual bids to enhance the value of FTRs has come under intense scrutiny in recent years [4].

In this paper, we consider a model of the day-ahead electricity market at any node in the network. Market participants include power generators and loads submitting physical bids, and financial players placing virtual bids. Virtual bids affect the day-ahead market clearing prices, but we assume that they have no impact on real-time prices. Theoretical results on interior Nash equilibria are given, assuming that virtual bidders can perfectly predict real-time prices and hold no FTRs sinking at the node. We then adopt a hypergame framework [5] to model the day-ahead market, assuming imperfect prediction of real-time prices by different virtual bidders. When no market participant holds FTRs, virtual bidders help achieve convergence between day-ahead and real-time nodal prices, as expected [6]. In this setting, we also allow one virtual bidder to hold an FTR position sinking at the node we consider (and sourcing at another node in the network, whose prices are taken as exogenous). We simulate FTR contracts of different size (10 MW and 30 MW). Our numerical results show that the larger the FTR position, the greater the incentive for the
FTR holder to place uneconomic virtual bids at the FTR sink to enhance the value of her financial position, in line with [7], [8].

We then discuss two mechanisms that could be used to detect a change in bidding behavior that affects market-clearing prices. In the first change-point detection mechanism, based on Kullback-Leibler divergence [9], we consider a sliding window of 15 observations. Assuming that day-ahead prices follow a Gaussian distribution, we estimate the distribution of prices inside each sampling window and calculate the KL divergence of this distribution from the distribution of prices under the null hypothesis of no change in bidding behavior. If the divergence measure is above a given threshold \( \lambda \), an anomaly in bidding behavior is detected. The second detection mechanism is based on the likelihood ratio test [10]. Also in this case, we test between the null hypothesis of no change in virtual bidding behavior and the alternative hypothesis of a change in bidding behavior to enhance the value of related financial positions. A change in the mean of the day-ahead price distribution is detected when the likelihood ratio between these two hypotheses, evaluated at the maximum likelihood estimate of the jump time in the sample, exceeds the threshold \( \lambda \).

In the absence of FTR positions, we assume that all market participants behave competitively and do not try to affect day-ahead prices: we denote as “competitive” a sample in which no player holds an FTR position. On the other hand, holding an FTR position provides an incentive to act strategically, trying to affect the day-ahead price to enhance the value of the contract: we define as “strategic” a sample in which one virtual bidder holds an FTR position. A trade-off exists between specificity and sensitivity, where specificity is defined as the percentage of competitive samples which are correctly considered competitive by regulators, relative to the total number of competitive samples, while sensitivity is the percentage of strategic samples which are correctly considered strategic by the regulator, relative to the total number of strategic samples [11]. By setting a higher threshold, we achieve higher specificity and lower sensitivity; conversely, if we set a lower threshold, we achieve high sensitivity but low specificity. Further, the larger the FTR position, the more abruptly day-ahead clearing prices will change, and the more easily anomalies in bidding behavior can be detected.

The rest of the paper is organized as follows. Section II describes player bids and market-clearing in the day-ahead energy market, while Section III provides theoretical results on interior Nash equilibria, assuming that virtual bidders do not own power generation units or serve load, and submit INC and DEC bids. Once made, bids are assumed revealed to all players.

In the following, real-time settlement is not modeled and the real-time price is taken as exogenous\(^1\). Thus, we assume that a sequence of real-time prices \( p_{RT}(t) \) is generated independently of day-ahead bidding, where \( t \) is integer-valued and hourly.

Assume players have linear bidding functions. Each trader makes a linear INC or a linear DEC bid. If trader \( i \in I \) (i.e., an INC bidder), her hourly bid is assumed of the form:

\[
q_{inc,i}(p; s_i) = (p - p_{min,i})^+ s_i
\]

where \( q_{inc,i} \) is the virtual supply at price \( p \), \( s_i \geq 0 \) and \( p_{min,i} \geq 0 \). If trader \( i \in D \) (i.e., a DEC bidder, where set \( D \) is disjoint from set \( I \)), her hourly bid is assumed of the form:

\[
q_{dec,i}(p; s_i) = (p_{max,i} - p)^+ s_i
\]

where \( q_{dec,i} \) is the virtual demand at price \( p \), \( s_i \geq 0 \), \( p_{max,i} > 0 \), and \( (z)^+ := \max\{z, 0\} \).

Each financial players can choose to place INC or DEC bids, and change the slope \( s_i \) of her INC or DEC curve. We assume that the maximum and minimum bidding prices, \( p_{max,i} \) and \( p_{min,i} \), are fixed and known to all other traders.

Since virtual bids typically represent less than 20% of scheduled energy [2], we introduce physical supply and demand linear curves. The physical curves are formulated as the sum of an inelastic component and a price-sensitive component, as follows\(^2\):

\[
Q_I(p) = (p - p_{min,I})^+ s_I + I_0
\]

\[
Q_D(p) = (p_{max,D} - p)^+ s_D + D_0
\]

where \( Q_I \) and \( Q_D \) denote physical supply and demand, respectively, and \( I_0 \) and \( D_0 \) represent the inelastic portions of the physical supply and demand curves. The parameters of equations (3) and (4) (i.e., \( s_I, s_D, p_{min,I}, \) and \( p_{max,D} \)) are non-negative, fixed and not affected by virtual bids.

Given \( D, I \), and \( \{s_i\} \), the day-ahead market clearing price \( p_{DA} \geq 0 \) is the solution \( p \) to:

\[
\sum_{i \in D} q_{dec,i}(p; s_i) + Q_D(p) = \sum_{i \in I} q_{inc,i}(p; s_i) + Q_I(p)
\]

The market clears at the intersection point of the demand and supply piecewise linear curves, and the quantity bid by player \( i \) that is cleared in the market is \( q_i(p_{DA}^*, s_i) \).

A cleared INC bid earns positive revenue if the day-ahead price is higher than the real-time price, while a DEC bid earns positive revenue if the real-time price is higher than the day-ahead price. In the absence of costs for placing virtual positions, each player’s revenue correspond to her profits. The profit of an INC bidder is given by:

\[
\Pi_{inc,i} = q_{inc,i}(p_{DA}^*, s_i)(p_{DA} - p_{RT}) \quad \forall i \in I
\]

\(^1\)The real-time market clears much smaller volumes of energy than the DA market, typically accounting for about 5% of scheduled energy (p.59, [12]).

\(^2\)Without physical generation and loads, and in the absence of any market imperfections such as asymmetric information, transaction costs, and risk aversion, the virtual traders would have no arbitrage opportunity.
while the profit of a DEC bidder is defined as:
\[
\Pi_{\text{dec,i}} = q_{\text{dec,i}}(p_{\text{DA}} - p_{\text{RT}} - p_{\text{DA}}) \quad \forall i \in D
\] (7)

III. NASH EQUILIBRIA ASSUMING PERFECT REAL-TIME PRICE PREDICTIONS AND NO FTRs

We start with a simple model of the day-ahead market with multiple virtual bidders who can perfectly predict the real-time price. Even for this simplified case, feasible solutions of the first-order necessary conditions for the Nash equilibrium [13] \((\forall i, \partial \Pi_i/\partial s_i = 0)\) are not available in closed form.

Claim 1. For a market with multiple virtual bidders, if all the bidders can perfectly predict real-time prices day-ahead, and the day-ahead clearing price is less than the real-time price \((p_{\text{DA}} = 0 \leq p_{\text{RT}})\), then at Nash equilibrium \((s^*)\), the day-ahead clearing price is less than or equal to the real-time price \((p_{\text{DA}} = 0 \leq p_{\text{RT}})\), and all the bidders will choose to submit DECs.

Proof: First note that all the virtual bidders receive zero revenue when \(s = 0\) irrespective of their roles (INC or DEC). To prove by contradiction, assume that at a Nash equilibrium \(s^*\) the DA clearing price is higher than the RT price. Since the DA price at \(s^*\) is higher than the DA price without virtual bidding \((s = 0)\), at least one of the bidders submits DEC bids at \(s^*\). Otherwise, if both bidders submit INCs, then \(p_{\text{DA}}\) will be lower than \(p_{\text{DA}} = 0\), as illustrated in Fig. 1. Since the DA price is greater than the RT price at \(s^*\), then according to the profit function of a DEC bidder defined in Section II (equation (7)), the bidder will incur a loss; hence, a zero bid is a better strategy. Therefore, \(s^*\) cannot be Nash - a contradiction. Since the Nash equilibrium DA clearing price is less than or equal to the RT price, the bidders must make DEC bids to receive non-negative profits.

We can show that if the DA market-clearing price is less than \(p_{\text{RT}}\), as assumed, \(s_i \geq 0, \forall i\), and \(\min_i \{p_{\text{max,i}}\} > p_{\text{RT}}\), then \(h_i, m_i < 0\) and \(g_i > 0, \forall i\). Therefore, if there exists a point \(s^*\) in the compact and convex set
\[
C \triangleq \{ s : p_{\text{DA}}(s) \leq p_{\text{RT}}, s_i \geq 0, \forall i \},
\] (9)
such that
\[
\frac{\partial \Pi_i}{\partial s_i} = 0, \quad \forall i
\]
then \(s^*\) is a Nash equilibrium point (also note that at \(s\) outside this convex set, at least one of the bidders receives negative revenue).

To prove that such a point exists, define a function \(\gamma : \mathbb{R}^n \rightarrow \mathbb{R}^n\),
\[
\gamma(s) = \left[ s_i + \lambda_i \frac{\partial \Pi_i}{\partial s_i} \right]
\]
for real \(\lambda_i \neq 0, \forall i\). Note that a fixed point \(s\) of \(\gamma\) satisfies \(\partial \Pi_i/\partial s_i = 0, \forall i\).

We can show that if \(s_i = 0\), then \(\partial \Pi_i/\partial s_i > 0\); and if \(p_{\text{DA}}(s) = p_{\text{RT}}\), then \(\partial \Pi_i/\partial s_i, \forall i\) are negative. Thus, Nash equilibrium cannot reside at the boundary. Moreover, since \(\partial^2 \Pi_i/\partial s_i^2, \forall i\) are bounded everywhere in set \(C\); then according to the mean value theorem \(\partial \Pi_i/\partial s_i\) is Lipschitz.

Note that Claim 1 and Corollary 1 can also be extended to the cases where \(p_{\text{RT}}\) is influenced by the day-ahead virtual bids, so that \(|p_{\text{DA}} - p_{\text{RT}}|\) decreases.

In the following, we will discuss the existence of an interior Nash equilibrium under certain conditions.

Claim 2. For a market with multiple virtual bidders, if all the bidders can perfectly predict real-time prices day-ahead, \(\min_i \{p_{\text{max,i}}\} > p_{\text{RT}}\) and \(p_{\text{DA}} = 0 \leq p_{\text{RT}}\), then there exists an interior Nash equilibrium point \(s^*\) such that \(p_{\text{DA}} = s^* < p_{\text{RT}}\).

Proof: According to Claim 1, virtual bidders choose to submit DECs at the Nash equilibrium point. Expanding the first-order necessary conditions for Nash equilibrium, we have
\[
\frac{\partial \Pi_i}{\partial s_i} = h_i(s) \frac{g_i(s) s_i + m_i(s)}{(s_i + s_D + s_I)^3}
\]
where
\[
h_i(s) = \sum_{j \neq i} (p_{\text{max},j} - p_{\text{max},i}) s_j + (p_{\text{max},D} - p_{\text{max},i}) s_D + (p_{\text{max},I} - p_{\text{max},i}) s_I + D_0 - I_0
\]
\[
g_i(s) = \sum_{j \neq i} (2p_{\text{max},i} - p_{\text{RT}} - p_{\text{max},j}) s_j + (2p_{\text{max},D} - p_{\text{RT}} - p_{\text{max},i}) s_D + (2p_{\text{max},I} - p_{\text{RT}} - p_{\text{min},i}) s_I + I_0 - D_0
\]
\[
m_i(s) = \sum_{j \neq i} (p_{\text{min},j} - p_{\text{RT}}) s_j + (p_{\text{max},D} - p_{\text{RT}}) s_D + (p_{\text{min},I} - p_{\text{RT}}) s_I + D_0 - I_0 (\sum s_j + s_D + s_I)
\]
(8)

Fig. 1: INC bids decrease the DA clearing price.

By a symmetric argument we get the following.

Corollary 1. For a market with multiple virtual bidders, if all the bidders can perfectly predict real-time prices day-ahead and \(p_{\text{DA}} = 0 \leq p_{\text{RT}}\), then \(p_{\text{DA}} = s^* \leq p_{\text{RT}}\) and all the bidders will choose to submit INCs.
continuous. Therefore, there exists \( \{ \lambda_i \} \) such that the continuous function \( \gamma : C \to C \). Existence of a Nash equilibrium then follows from Brouwer’s fixed-point theorem.

By a symmetric argument we get the following.

**Corollary 2.** For a market with multiple virtual bidders, if all the bidders can perfectly predict real-time prices day-ahead, \( \max_i \{ p_{\text{min},i} \} < p_{\text{RT}} \) and \( p_{\text{DA}} |_{s^*} \geq p_{\text{RT}} \), then there exists an interior Nash equilibrium point \( s^* \) such that \( p_{\text{DA}} |_{s^*} > p_{\text{RT}} \).

To illustrate Claim 2, consider the example where:

\[
\begin{align*}
p_{\text{min},1} &= 0 & p_{\text{max},2} &= 17 \\
p_{\text{max},1} &= 20 & s_D &= 12 \\
I_0 &= 10 & s_I &= 10 \\
I_0 &= 15 & p_{\text{RT}} &= 15
\end{align*}
\]

so that the DA market-clearing price is lower than \( p_{\text{RT}} \), and \( \min \{ p_{\text{max},1}, p_{\text{max},2} \} > p_{\text{RT}} \). The normalized quiver plot of \( (\partial \Pi_1/\partial s_1, \partial \Pi_2/\partial s_2) \) showing an interior equilibrium point is given in Fig. 2.

![Quiver plot](image)

Fig. 2: Quiver plot of \( (\partial \Pi_1/\partial s_1, \partial \Pi_2/\partial s_2) \) with an interior equilibrium point

However, if \( p_{\text{RT}} \geq \min_i \{ p_{\text{max},i} \} \), then the set \( C \) of (9) is unbounded and therefore not compact, and an interior equilibrium point may not exist.

**IV. GENERATION OF TRADER BID ASSUMING KNOWN PRICE-FORECASTING CORRELATIONS**

In this section, we assume that virtual bidders cannot perfectly predict the real-time price. Different traders have different abilities to day-ahead forecast real-time electricity prices, as reflected by different variance parameters \( \varepsilon_i \), and will thus make different (unbiased) predictions for the real-time price.

The market is modeled by the following Monte Carlo hypergame framework.

For every hour \( t \):

1) the real-time price \( p_{\text{RT}}^t \) is generated based on a certain model.

2) each trader \( i \) makes her prediction of the RT price, \( p_i \), which can be modeled as \( N(p_{\text{RT},i}, \varepsilon_i) \), where \( N(p_{\text{RT},i}, \varepsilon_i) \) is an independent Gaussian random variable with mean \( p_{\text{RT}} \) and variance \( \varepsilon_i \), and determines her role (INC or DEC) and DA bid-slope \( s_i \) from the function \( \text{bids}(p_i, i, \varepsilon_{-i}) \) described below.

3) determine the day-ahead settlement price, \( p_{\text{DA}}^* \).

4) determine each trader’s profits and losses.

The function \( \text{bids}(p_i, i, \varepsilon_{-i}) \) works as follows:

1) model trader \( i \)'s belief of \( j \)'s real-time forecast as \( P_j \sim N(p_i, \varepsilon_j) \) for all \( j \neq i \).

   a) initialize \( \hat{s}_{\text{inc},i} = \hat{s}_{\text{dec},i} = 0 \), and iteration index \( k_{\text{inc}} = k_{\text{dec}} = 0 \).

   b) while \( k_{\text{inc}} \) and \( k_{\text{dec}} \) both below a count threshold (i.e., estimates lack sufficient confidence) do:

   i) generate \( p_{\text{RT}} \) according to 1).

   ii) initialize \( \hat{s} \) to 0 and \( \hat{s}^{(\text{prev})} \) to an arbitrary vector very different from 0.

   iii) while \( |\hat{s} - \hat{s}^{(\text{prev})}| \) is sufficiently large or there are any INC/DEC changes:

   A) \( \hat{s}^{(\text{prev})} = \hat{s} \)

   B) for each trader \( j \) (including \( i \)): decide to submit INCs or DECs, and compute her next \( \hat{s}_j \) through the following equation:

   \[
   \hat{s}_j(\text{next}) = \arg\max_{s} \{ \Pi_{\text{inc},j}(p_{\text{DA}}(s), s), \Pi_{\text{dec},j}(p_{\text{DA}}(s), s) \},
   \]

   where \( p_{\text{DA}} \) is a function of \( s \) [14]. For an INC bidder, \( p_{\text{DA}} \) is the price \( p \) that solves:

   \[
   \sum_{k \in D} q_{\text{inc},k}(p; s_k^{(\text{prev})}) + Q_D(p) = \sum_{k \in I \setminus j} q_{\text{inc},k}(p; \hat{s}_k^{(\text{prev})}),
   \]

   while for a DEC bidder, \( p_{\text{DA}} \) is the price \( p \) that solves:

   \[
   \sum_{k \in I \setminus j} q_{\text{inc},k}(p; s_k^{(\text{prev})}) + Q_D(p) = \sum_{k \in D \setminus j} q_{\text{dec},k}(p; \hat{s}_k^{(\text{prev})}),
   \]

   C) \( \hat{s} = \hat{s}_j(\text{next}) \)

   iv) if bidder \( i \) decides to submit INCs, \( k_{\text{inc}} + + \), update the average\(^4\) strategy \( \bar{s}_{\text{inc},i} \); else, \( k_{\text{dec}} + + \), update \( \bar{s}_{\text{dec},i} \).

2) if \( k_{\text{inc}} \geq k_{\text{dec}} \), bidder \( i \) chooses to submit INCs, return \( \bar{s}_{\text{inc},i} \); else, bidder \( i \) chooses to submit DECs, return \( \bar{s}_{\text{dec},i} \).

\(^3\) \( \varepsilon_{-i} \) represents the vector of variance parameters of players different than \( i \). We assume that bidder \( i \) can infer these based on the observation of the other participants’ past bids.

\(^4\) Note that there are other ways to update the strategy after \( T \) hours. For example, we can use bidder \( i \)'s revenue as weight so that \( \hat{s}_i = \frac{1}{T} \sum_{k=1}^{T} \Pi_{\text{inc},j}(p_i, s_k, \bar{s}_{-i}) \), where \( \Pi_{\text{inc},j}(p, s, \bar{s}) \) is bidder \( i \)'s revenue and strategy at the \( k \)th hour. Alternatively, we can calculate the strategy as \( \hat{s}_i = \frac{1}{T} \sum_{k=1}^{T} \Pi_{\text{inc},j}(p_i, s_k, \bar{s}_{-i}) \), where \( \bar{s} \) is the average of other bidders’ strategies over \( T \) hours, and \( \Pi_{\text{inc},j}(\cdot) \) is either the revenue function \( \Pi_{\text{inc},j} \) or \( \Pi_{\text{dec},j} \) defined at the end of Sec. II.
To determine her $s_i$ at each time $t$, each trader $i$ independently simulates a day-ahead bidding game involving all traders, and averages the Nash equilibrium play actions (alternatively, averages could be weighted based on computed revenues of trader $i$). Therefore, the above framework is similar to that of hypergames [5].

A. Possible manipulation in FTR position

When the power grid is congested, bidders will need to pay Transmission Congestion Charges and the price differences in Day-ahead Congestion Prices [15]. A Financial Transmission Right (FTR) is used to compensate the financial loss of its holders in the congested grid, and therefore to hedge the risk of congestion-driven price increases [8].

Suppose congestion happens on the transmission line from source to sink. Accordingly, the congestion price at the source ($CP_{source}$) declines, while the congestion price at the sink ($CP_{sink}$) increases. To compensate the financial loss due to the price difference, a bidder holding $F$ MW FTR at the sink will receive ($CP_{sink} - CP_{source}$) $\times F$ compensation. As a result, when the grid is congested, FTR could be another source of income and therefore may skew the strategy of those aware holders. If the FTR is large enough, a virtual bidder’s exploitations of this position will cause the DA and RT prices to diverge, offsetting the merit (convergence between DA and RT prices) brought by the virtual bidding. We will observe this effect in Sec. V-B.

V. Numerical experiments

A. No market participant holds FTR positions

In this section we test our model, assuming that no market participant holds FTR contracts. We set the slopes of physical supply ($s_1$) and demand ($s_2$) to 10 MW/$ and 12 MW/$, $I_0$ and $D_0$ to 10 MW and 15 MW, $p_{min}$ and $p_{max}$ to 0 MW and 20 MW, and $I_0$ and $D_0$ to 10 MW and 15 MW. Given these parameters and in the absence of virtual bidding, the hourly DA market clearing price is 11.1364 $/MWh. The real-time price is assumed equal to 12 $/MWh, a little higher than the clearing price; this provides DA/RT spread on which virtual bidders can arbitrage. We introduce two bidders in the DA market. These bidders can choose to submit INC or DEC bids, and select the slope of their bidding function $s_i$. Let $p_{min,1}$ and $p_{min,2}$ be 5 MW/$ and 7 MW/$, and $p_{max,1}$ and $p_{max,2}$ be 16 MW/$ and 17 MW/$. Initially, bidders do not bid anything, i.e., $[s_1, s_2] = [0, 0]$.

In general, each virtual bidder simulates the hourly day-ahead bidding game as described in Section IV (i.e., runs the function bids($p_{i,t,\epsilon}$) independently and bids accordingly). Each player makes a real-time price prediction at each hour in our experiment; given different variance parameters, real-time price predictions will differ by player at each hour.

First, assume $\epsilon_1 = 0.02$ and $\epsilon_1 = \epsilon_2 = : \epsilon_2$. Bidder 1 predicts the RT price to be 12.12 $/MWh, and is assumed to select her strategy according to the function bids($p_{1,t,\epsilon}$).

The convergence of $\bar{s}_1$ over 1000 iterations in a single hour characterized by a RT price prediction of 12.12 $/MWh by all players is shown in Fig. 3. After hundreds of experiments, the average slope of bidder 1’s virtual demand function, $\bar{s}_1$, stabilizes around 1.7 MW/$.

Next, suppose the real-time price is equal to 12 $/MWh over the next 49 hours, $\epsilon_1 = 0.02$ and $\epsilon_2 = 0.01$: thus, although both bidders can make fairly accurate RT price predictions, bidder 2’s estimates are more precise. At every hour, each virtual bidder selects her strategy according to the function bids($p_{1,t,\epsilon}$). The simulated profits for the both bidders over the 50 hours are shown in Fig. 4. As expected, bidder 2, who can estimate the RT price more accurately, earns higher profits on average. We also observe that, due to the possibility of profiting from DA/RT price spreads, the DA market-clearing price is closer to the RT price, relative to the case in which no virtual bidding is allowed and the DA price is obtained from the intersection of physical supply and demand. Thus, virtual bidding helps improve convergence of DA and RT prices in wholesale electricity markets [6].

B. Virtual bidder 1 holds FTR positions sinking at the node

In this section we extend our model to demonstrate the effect of FTRs on the day-ahead energy market. As noted in Section I, FTRs are financial contracts that entitle their holders the right to collect a payment when day-ahead congestion occurs in the direction specified in the contract (i.e., from the source node to the sink node). The hourly value of an FTR is proportional to the difference between day-ahead price at the sink and day-ahead price at the source of the contract. In our
experiment, we continue to assume that the real-time price is equal to 12 $/MWh, and set the variance parameters for the two virtual bidders to $\varepsilon_1 = 0.03$ and $\varepsilon_2 = 0.05$. During the first 50 hours of our simulation, the game proceeds as described in the previous section: that is, virtual bidders select their optimal strategy without considering FTR revenues. After the 50th hour, bidder 1 acquires an FTR position sinking at the node we consider in our model. The day-ahead price at the FTR source is assumed to be 11. We simulate the effect of FTR contracts of different size: 10 MW and 30 MW. Figures 5 and 6 show the bidders’ profits (excluding FTR revenues) and DA market-clearing prices for a 10 MW FTR, while Figures 7 and 8 illustrate these results for a 30 MW contract.

Fig. 4: Bidders’ profits over 50 hours.

Fig. 5: Bidders’ profits over 80 hours. After 50 hours, bidder 1 starts holding a 10 MW FTR.

After bidder 1 acquires her FTR position, energy-market profits of both bidders decline, while the DA market-clearing prices increase. The reason is that the FTR creates an incentive for its holder to place DEC bids at the contract’s sink. Under the assumption that these DEC bids clear in the DA market, the DA prices tend to increase over time, and the gap between DA and RT prices at the sink widens (with the DA prices that are higher than RT prices), making virtual bids uneconomic on a stand-alone basis. Although the strategy is profitable for bidder 1, who offsets losses on the energy positions with larger gains on the FTR positions, virtual bidder 2 is negatively affected and makes lower profits on her energy positions as a result.

Further, the larger the FTR position, the greater the incentive for the FTR holder to engage in uneconomic virtual bidding behavior. Thus, when we simulate a 30 MW FTR position DA prices are higher on average, and losses on the virtual positions are larger for both financial players.
VI. DETECTING ANOMALIES IN VIRTUAL BIDDING BEHAVIOR IN DAY-AHEAD ELECTRICITY MARKETS

In this section, we discuss two methods to detect a change in bidding behavior that affects day-ahead market-clearing prices. The first method is based on Kullback-Leibler divergence, while the second relies on a likelihood ratio test.

In the following, we assume that the characteristics of the day-ahead electricity market and market participants remain the same over the period of analysis. Therefore, statistically significant changes in the statistical properties of market-clearing prices can only be attributed to changes in bidding behavior of existing market participants, and only virtual bidders may have an incentive to act strategically. As a result, regulators can detect anomalies in virtual bidding behavior by monitoring the day-ahead market-clearing price.

A. Kullback-Leibler divergence

Assume we know the statistical properties (e.g., mean and variance) of the day-ahead market-clearing price without FTRs. We can use Kullback-Leibler divergence to detect the deviation of the statistics of samples in the current sampling window from the a priori clearing price model. We use Gaussian distributions to model the clearing prices. Assume that in the absence of FTRs the day-ahead market-clearing price has Gaussian density $f_0(x|\mu_0, \sigma_0^2)$. To detect a change in bidding behavior (given FTR positions that are unknown to market participants other than the holders), we calculate the mean ($\mu$) and variance ($\sigma^2$) of samples defined by a sliding window length of 15, and construct the Gaussian pdf $f(x|\mu, \sigma)$. The Kullback-Leibler divergence of $f$ from $f_0$ is (assuming $f_0$ is the “target” position):

$$D_{KL}(f_0||f) \triangleq \int_{-\infty}^{\infty} f_0(x) \log \frac{f_0(x)}{f(x)}$$

$$= \log \frac{\sigma}{\sigma_0} + \frac{(\mu_0 - \mu)^2}{2\sigma^2} + \frac{\sigma_0^2}{2\sigma^2} - \frac{1}{2}$$

The Kullback-Leibler divergence is presented in Fig. 9 for the two cases in which the virtual bidder holds FTR positions of 10 MW and 30 MW, respectively.

![Fig. 8: DA prices over 80 hours. After 50 hours, bidder 1 starts holding a 30 MW FTR.](image)

![Fig. 9: Kullback-Leibler divergence for the day-ahead price when bidder 1 holds a 10 MW or 30 MW FTR after the 50th hour.](image)

By setting a threshold $\lambda$, we are able to detect a change in bidding behavior affecting clearing prices according to:

$$D_{KL} \leq \lambda$$

$H_0$: No-FTR hypothesis.

$H_1$: FTR hypothesis.

For example, if we set $\lambda = 0.2$, we will detect uneconomic bidding at hour 62 when the FTR position is equal to 10 MW, and hour 52 when the FTR position is equal to 30 MW, as shown in Fig. 9. We can increase the detection sensitivity by setting a lower $\lambda$, but the probability of false alarm would accordingly increase. Another way to trade off between specificity and sensitivity is by setting a reasonable window size. If the window size is small, due to the short “memory” of our detection system, detection will occur faster. On the other side, however, the divergence measure will be noisier, potentially detecting a change too often.

B. Likelihood ratio test

Detection based on a likelihood ratio (LR) test [10] detects an abrupt change of mean value, with no need to select a window size. The null and alternative hypotheses are defined as above. The Gaussian density distribution of day-ahead
prices under the null hypothesis is \( f_0(x) \), with mean \( \mu_0 \), while the density distribution under the alternative hypothesis is \( f(x) \), with mean \( \mu \). \( f(x) \) and \( f_0(x) \) are assumed to have the same variance. The goal of this detection measure is to identify a change in the mean of the distribution and the time \( r \) at which the change occurs. Given a sample of length \( n \) (where \( n \) is equal to 80 in our case), we try to find the optimal change point \( \hat{r} \) in our sample using the likelihood ratio so that:

\[
\hat{r} = \arg \max_{1 \leq r \leq n} \prod_{k=r}^{n} \frac{f(p_{DA}^{(k)})}{f_0(p_{DA}^{(k)})}, \tag{11}
\]

where \( f(x) \) is the density distribution of day-ahead prices under the alternative hypothesis, while \( f_0(x) \) is the pdf under the null hypothesis. Taking the log of Eq. (11), we have:

\[
\hat{r} = \arg \max_{1 \leq r \leq n} \sum_{k=r}^{n} (p_{DA}^{(k)} - \mu_0 - \frac{v}{2}) \tag{12}
\]

where \( v = \mu - \mu_0 \) is the magnitude of the jump. If \( \mu_0 \) is not known a priori, it can be estimated as \( \mu_0 = \sum_{k=1}^{r} p_{DA}^{(k)} / r \).

We detect a jump in the mean at the first time \( n \) at which the change detector \( g_n \) exceeds a given threshold \( \lambda \), that is:

\[
g_n = \max_{1 \leq r \leq n} \sum_{k=r}^{n} (p_{DA}^{(k)} - \mu_0 - \frac{v}{2}) \leq \lambda. \tag{13}
\]

In practice, we of course also do not know the mean clearing price \( \mu \) under the alternative hypothesis of change in day-ahead mean prices. [10] propose a modified approach to solve this problem, which replace the jump magnitude with an additional variable, \( v \):

\[
\max_{v} \max_{r} \sum_{k=r}^{n} (p_{DA}^{(k)} - \mu_0 - \frac{v}{2}) \leq \lambda \tag{14}
\]

where the inner "max" can be solved by using the first order optimality condition with respect to \( v \), so that the optimized choice for \( v \) is given by:

\[
\hat{v} = \frac{1}{n - r + 1} \sum_{k=r}^{n} (p_{DA}^{(k)} - \mu_0) \tag{15}
\]

The likelihood ratios for the cases in which virtual bidder 1 holds an FTR of 10 MW and 30 MW are shown in Fig. 10.

In choosing the threshold \( \lambda \), we balance specificity and sensitivity. For example, if we set \( \lambda = 4 \), we would detect uneconomic bidding at 55 and 52 for FTR = 10 MW and FTR = 30 MW, respectively, in Fig. 10.

VII. SUMMARY

We consider a model of a day-ahead electricity market under the simplifying assumption that real-time prices are not affected by virtual bidding. Theoretical results on interior Nash equilibria are given, assuming virtual bidders can make perfect prediction of real-time prices and hold no Financial Transmission Rights. A hypergame framework is then used to model the day-ahead market assuming imperfect prediction of real-time prices by different virtual bidders with different accuracies. Numerical results demonstrate the effect on the hypergame of FTR positions that are unknown to all market participants except the holder, and how to detect their presence.

Fig. 10: LR test for the day-ahead price when virtual bidder 1 holds a 10 MW or 30 MW FTR after the 50th hour.