On Fair Attribution of Costs Under Peak-based Pricing to Cloud Tenants

Technical Report #15-002

Abstract

The costs incurred by cloud providers towards operating their data centers are often determined in large part by their peak demands. The pricing schemes currently used by cloud providers to recoup these costs from their tenants, however, do not distinguish tenants based on their contributions to the cloud’s overall peak demand. Using the concrete example of peak-based pricing as employed by many electric utility companies, we show that this “gap” may lead to unfair attribution of costs to the tenants. Simple enhancements of existing cloud pricing (e.g., analogous to the coincident peak pricing (CPP) used by some electric utilities) do not adequately address these shortcomings and suffer from short-term unfairness and undesirable oscillatory price vs. demand relationship offered to tenants. To overcome these shortcomings, we define an alternative pricing scheme to more fairly distribute a cloud’s costs among its tenants. Our approach to fair attribution of cloud’s costs is inspired by the concept of Shapley values used to fairly divide revenue among participants of a financial coalition. We demonstrate the efficacy of our scheme under price-sensitive tenant demand response using a combination of (i) extensive empirical evaluation with recent workloads from commercial data centers operated by IBM, and (ii) analytical modeling through non-cooperative game theory for a special case of tenant demand model.

Keywords

Cloud Tenant; Pricing Design; Game; Fairness

I. INTRODUCTION

Cloud computing is turning information technology (IT) into a utility wherein cloud providers hide the complexity of building and operating data centers from their customers (“tenants”) and supply them with virtualized IT resources. Inevitably, this virtualization creates a “gap” between how cloud providers incur costs for operating their data centers and how they recoup these costs from their tenants, the latter typically in terms of virtualized IT resources such as virtual machines (VMs). One important example of this gap is the difference that often exists between pricing schemes underlying costs incurred by cloud providers versus those used by cloud providers themselves to sell virtualized IT resources.

The Problem: Several important components of a cloud provider’s costs are significantly affected by the peak of its resource usage. Arguably, the most explicit example of this - our focus in this paper - is found in the form of electric bills using “peak-based pricing” that certain electric utility companies employ, e.g., [1], [2]. In this pricing, the consumer’s electric bill contains a separate component for its peak power draw over the billing cycle (“peak charge”) in addition to the usual component based on energy consumed (“energy charge”). It is well-known that the peak-related component can contribute a lot to the overall electric bill of the cloud provider [3], [4].
Whereas demand response (DR) for optimizing costs under such pricing has received a lot of attention recently [5], [6], [4], we identify a novel and complementary concern that we find useful to think of as a “fairness” problem. Notions of fair pricing of tenants in our problem are more complex than simple notions of proportional or max/min fairness due to a plurality of quantities used (mean and peak) for pricing and the complex statistics of the coincident/aggregate peak itself.

Our problem is most clearly understood by posing the following question in a revenue-neutral cloud environment: how should a cloud provider subject to peak-based pricing for its electric power consumption recoup these costs from its tenants? Whereas the answer is straightforward for the energy charge making up the bill (simply charge each tenant for its own total energy consumption), it becomes less clear when one considers recouping the peak charge. It is easily seen that existing pricing schemes employed by cloud providers amount to distributing the peak charge among tenants in proportion to their resource allocations (e.g., VMs) or, as a first-order approximation, in proportion to their energy consumption. We find such a pricing scheme to be “unfair” since it charges two tenants identically even if one of them contributes more to the cloud’s overall peak power draw than the other (Figure 2 shows a simple example). In fact, as we show in Section II, even other pricing schemes that do incorporate the tenants’ contributions to the cloud’s peak draw into their decision-making continue to suffer from such unfairness. This motivates us to explore an alternative pricing scheme that attributes peak-related costs more carefully leading to fairer charging of tenants.

Why Study This Problem? We find our problem worth studying for two main reasons. First, energy-related costs are already significant components of overall costs for many data centers, and correspondingly of their tenants. Consider the following excerpt from a recent New York Times article on how certain data centers already bill their tenants: “... electrical capacity is often the central element of lease agreements, and space is secondary. A result, an examination shows, is that the industry has evolved from a purveyor of space to an energy broker - making tremendous profits by reselling access to electrical power, and in some cases raising questions of whether the industry has become a kind of wildcat power utility ... 75 percent of Resurgens’s lease was directly related to power - essentially for access to about 30 power sockets.” [7] It is likely that the relative contribution of energy costs will only continue to grow due to increasing energy prices, especially those for peak draw.

Second, one may wonder if our fairness concern can be alleviated simply by replacing peak-based pricing with real-time pricing. In fact, many providers and consumers do find peak-based pricing more appealing due to the lower associated uncertainty compared to spot prices. As evidence that such choices are already being made by existing data centers and their tenants, consider the following excerpt from the same New York Times article: “One key to the profit reaped by some data centers is how they sell access to power. ... many data centers charge tenants as if they were using all of that capacity - in other words, full price for power that is available but not consumed.” Generally speaking, since spot prices can be highly dynamic, it may be desirable for certain energy consumers to hedge their electricity costs by buying from a third party an energy future whose form may be very similar

1A revenue-neutral cloud ensures that the costs it recoups from its tenants are exactly what it incurs due to its own operation.
to peak-based pricing, or to explore a trade-off between higher upfront costs vs. lower variability in subsequent spot prices. The choice of an electricity pricing structure for a cloud provider depends on its demand properties (predictability, flexibility, variability, etc) as well as the variation of the spot price, compared with the hedged rate and the peak penalty. As an example, a data center whose demand is not flexible and less variable might choose a flat rate with peak power penalty; whereas another data center with highly flexible demand, might prefer real-time energy price, if the price variation is large enough for it to do arbitrage.

**A Note on Generality:** Although we focus only on energy-related operational expenses, there are several other data center costs that also have a peak usage-based component (although perhaps more indirectly). Internet Service Providers (ISPs) often employ tariff schemes for bandwidth, based on a high percentile that closely resemble peak charging (e.g., 95th or 99th percentile of the empirical distribution of bytes sent per measurement window over the billing cycle). In certain multi-homed settings, a cloud provider may employ a mixture of ISPs, some that charge based on raw bytes sent (like energy charge) while others based on a high percentile (similar to peak charge) [8]. Finally, many cloud providers (e.g., public clouds such as Amazon EC2 [9]) are perhaps best modeled as being interested in profit maximization, and the fairness concerns we discuss may not be meaningful for them. However, there are many cloud computing environments where fair cost attribution may be a valid concern. Private clouds catering to departments/groups within an enterprise are an example.

**Contributions:** For a future revenue-neutral cloud’s operational costs under peak-based pricing, we define an alternative pricing scheme to more fairly distribute these costs among its tenants which is inspired by the concept of Shapley values [10] that is used to fairly divide revenue among participants of a financial coalition. Our contributions are both in terms of empirical performance evaluation methodology and theoretical results. Our proposed pricing scheme uses the first and second order statistics of tenant’s workloads as an effective proxy for tenants’ contribution to the cloud peak. This pricing is considered for cases without and with tenants’ engagement in DR. Particularly, we studied the case when tenants actively engage in DR to maximize their net utility by shedding demand. We empirically evaluate both cases, using stationary and non-stationary workloads (the former synthesized from the latter) from IBM production data centers. Here is a summary of our contributions:

- The performance of this pricing scheme is empirically compared against a baseline scheme through the use of recorded tenant demands from a real-world commercial data center (IBM), made challenging by the highly variable and non-stationary aspects of the workload data.
- The alternative pricing scheme under demand response (DR) is studied analytically through non-cooperative game theory for a special case of tenant demand model. We show a kind of “incentive compatibility” result where tenants with higher contributions to aggregate demand variation (and hence to peak charges incurred by the cloud) pay more.

**Outline:** The rest of this paper is organized as follows. In Section II, we present some background and motivation using some straw-man pricing schemes. In Section III, we define our alternative pricing scheme and carry out an empirical evaluation of its efficacy. In Section IV, we explore the impact of tenant demand-response (by demand
shedding) by game-theoretic analysis and through empirical evaluation using real-world workload traces. We discuss related work in Section V and identify directions for future work in Section VI.

II. BACKGROUND AND MOTIVATION

We assume that our cloud provider procures electric power from an electric utility company that it uses to power its data centers (both IT equipment and non-IT infrastructure like cooling). We will use the terms “demand” and “power” interchangeably throughout the paper. The cloud provider pays the electric utility company a bill at the end of $d^{th}$ billing period of discrete length $K$ that has the following form:

$$ P_d = \alpha K M_d + \beta X(k^*(d)),$$

where $\alpha$ and $\beta$ are the energy price and peak power price from the electric utility in units of $$/kWh and $$/kW, respectively. $X(k)$ denotes the cloud’s power consumption during the $k^{th}$ time slot, $M_d = \frac{1}{K} \sum_{k=1+K(d-1)}^{Kd} X(k)$ is the mean power consumption over the $d^{th}$ billing period, and $k^*(d) = \arg \max_k \{X(k), 1+K(d-1) \leq k \leq Kd\}$ is the time slot in which the cloud’s peak power demand over this billing cycle occurs. Figure 1 shows an example of such pricing. This is a simplified form of the tariff scheme employed by several electric utility companies for their large consumers. Although we choose to work with this specific pricing scheme between the utility and the data center, the problems we identify and the insights we develop likely apply more generally. For example, instead of peak-based pricing, much shorter term (e.g., day ahead or hour ahead) spot prices are often employed by electric utilities [11], [12].

Since such prices can be highly dynamic, it is natural for the consumer to hedge its electricity costs based on its demand properties (e.g., less variation over time) and to avoid highly uncertain spot prices. This could be through some third party entity or directly from power utility company which offers (presumably lower) flat rate price and a peak penalty. The cloud provider in turn presents tenant $i$ with a bill $p_{i,d}$ where $1 \leq i \leq N$ and all $N$ tenants are “long-lived” in the sense of existing for the entire billing period (or several such periods). We discuss “short-lived” or “fleeting” tenants momentarily. We assume revenue neutrality, i.e., $P_d = \sum_i p_{i,d}$.

Our interest is in notions of fairness in how the provider divides $P_d$ into $p_{i,d}$.

We make the simplifying assumption that the cloud provider has accurate power metering and accounting techniques which it can accurately partition its overall power consumption $X(k)$ during the $k$-th time slot into $x_i(k)$, the contribution of the $i$-th tenant. This is a complementary problem on power metering of the VMs which uses utilization of resources such as CPU, memory and disk usage by the VMs [13]. We assume the solution exists for power metering and calculating each VM contribution to the whole power consumption. We discuss three intuitively appealing “straw-man” pricing schemes and their pros and cons. Our discussion of these baseline pricing schemes helps us identify desirable features that we would like our alternative pricing scheme to possess. We propose such an alternative pricing scheme in Section III-A.
Existing Pricing: In current cloud environments, tenants are charged based on their usage/allocation of virtualized IT resources such as VMs without distinguishing contributions to the cloud’s peak demand. Our first baseline represents such pricing (hence we call it “existing pricing”) and operates by dividing $P_d$ among tenants in proportion to their mean demands over the billing cycle. That is,

$$p_{i,d}^1 = \alpha K \mu_i(d) + \beta X(k^*(d)) \frac{\mu_i(d)}{\sum_j \mu_j(d)}$$

(2)

where $\mu_i(d)$ is tenant $i$’s mean power demand over the $d$-th billing period. To appreciate a key shortcoming of this scheme, consider the example shown in Figure 2 where both tenants have the same mean demand $\mu$ but have different demand variations (and hence different contributions to the cloud’s overall peak power consumption). Existing pricing results in both these tenants being charged identically. However, one might find this unfair since tenant 2 contributes much more to the peak-based component of the cloud’s overall costs than tenant 1.

Local-Peak based: A second baseline scheme charges each tenant in proportion to its own “local” peak that occurs during time slot $k^*_i(d) = \arg \max_k \{ x_i(k), 1 + K(d - 1) \leq k \leq Kd \}$ for the $i$-th tenant,

$$p_{i,d}^2 = \alpha K \mu_i(d) + \beta X(k^*(d)) \frac{x_i(k^*_i(d))}{\sum_j x_j(k^*_j(d))}.$$  

(3)

This pricing scheme may seem appealing at first glance in that it allows a tenant to feel “isolated” from others. However, such isolation holds only when all the tenants’ local peaks occur together. In practice, different tenants may peak at different times, either due to inherent workload properties or due to their DR. In such cases, local peak-based pricing may be unfair to tenants that peak in a way that doesn’t contribute to the overall peak. Equally problematic is that this pricing has the potential of not discouraging DR by load shifting that actually worsens the peak of the cloud’s overall demand.

Contribution to Actual Peak: Alternatively, the cloud could split the peak power costs among tenants according to their contribution to the its overall peak. That is,

$$p_{i,d}^3 = \alpha K \mu_i(d) + \beta X(k^*(d)) \frac{x_i(k^*(d))}{\sum_j x_j(k^*(d))}.$$  

(4)

This closely resembles the “coincident peak pricing (CPP)” employed by many electric utilities wherein they employ a higher-than-usual energy price during periods of high aggregate demand [14], [15]. Similar to CPP, one simple implementation in our ecosystem could be based on the cloud sending warnings of possible coincident peaks and the tenants incorporating these warnings into their DR. One expects this pricing scheme to fairly reflect the tenant’s contributions to peak power in the long term (i.e., lasting multiple billing periods). That is, in an asymptotic sense, this scheme remedies the fairness problems of the last two baselines. Consequently, we will use this scheme as the baseline against which we will compare our proposal in the remainder of the paper.
This baseline continues to exhibit fairness problems in the short term. For a given billing period a particular tenant may “by chance” have an unusually large $x_i(k^*)$. Such short-term unfairness may be undesirable for (i) charging “fleeting” tenants that procure resources from the cloud provider over short periods (relative to the billing duration), and (ii) if workload characteristics change frequently/abruptly (as we will demonstrate in Section III-B).

III. AN ALTERNATE PRICING SCHEME

A. Arguments and Design

The previous discussion helps us identify two desirable features we would like to see in our alternative pricing scheme. First, it should fairly incentivize tenant behavior that reduces overall costs. E.g., if a tenant tends to reduce its offered demand when an overall peak occurs (either inherently or via its DR), its costs should be lower than if it behaved otherwise. Second, we would like it to offer more stable demand vs. price relation to a tenant than the baseline does - tenants tend to prefer lower price fluctuations between billing periods as within billing periods. E.g., a tenant with roughly the same demand during two different time slots should not see very different prices as this would complicate its decision-making. The baseline is susceptible to such behavior due to atypical events involving coincident peak occurrences only by chance.

We find two complementary design guidelines useful in achieving these desirable features.

- **Guideline A:** Our pricing scheme should fairly incorporate into a tenant’s costs an explicit measure of how this tenant’s demands contribute to the costs of the cloud, e.g., this measure should be a reasonably accurate proxy for contribution to the cloud’s peak. That is, tenant pricing should be similar to how Shapley values divide among participants the revenue gained by their coalition [10].

- **Guideline B:** A tenant’s peak-related costs should be determined not merely by its contribution to the most recent billing period’s peak demand (or other such potentially only “chance events”) but rather by a statistical measure that can be assessed with greater confidence.

We develop our alternate pricing policy with assumptions of workload stationarity for all the tenants. Subsequently, we consider the adaptation of these basic ideas to pricing for tenants with more complex real-world workloads that are not stationary. Dropping the subscript $d$ in the notation from Section II (based on our stationarity assumption), recall that over a given billing period of (discrete) time of length $K$, we denote as $x_i(k)$ the demand at time $k \in \{1, 2, ..., K\}$ of tenant $i$, $i \in \{1, ..., N\}$. Let us denote its mean as $\mu_i = \mathbb{E}x_i(k)$ and variance $\sigma^2_i = \mathbb{E}(x_i(k) - \mu_i)^2 \forall k$. Let us also assume that the demands at the same time $k$ of tenants $i$ and $j$, $x_i(k)$ and $x_j(k)$, are correlated, and define

$$c_{i,j} = \mathbb{E}((x_i(k) - \mu_i)(x_j(k) - \mu_j))$$

As such, the cloud’s aggregate demand $X := \sum x_i$ has mean $M = \sum \mu_i$ and variance $S^2 = \mathbb{1}^T C \mathbb{1}$, where the co-variance matrix $C = [c_{i,j}]$, $\mathbb{1}$ is a $N$-vector of 1’s. In this manner our model accounts for correlated demand variation among the tenants at the same time, and thus provides a simple way to explain “coincident peak” demands.
To define an alternative pricing policy, first define the \((N - 1) \times (N - 1)\) co-variance matrix \(C_{-i}\) without the tenant \(i\) and let \(\mathbb{1}_{-i}\) be the \((N - 1)\)-vector of 1’s. Also consider the aggregate demand variance without tenant \(i\) (based on guideline A above), i.e., the variance of \(X - x_i\),

\[
S_{-i}^2 = \mathbb{1}_{-i}^T C_{-i} \mathbb{1}_{-i}.
\]

So, tenant \(i\)’s contribution to the variance of the aggregate demand \(X\) is

\[
S_i^2 = S^2 - S_{-i}^2.
\]

Let \(G(\mu, \sigma)\) be the expected peak over the billing period \(\{1, 2, ..., K\}\) for the demand process parameterized by mean \(\mu > 0\) and standard deviation \(\sigma > 0\) (based on guideline B above). Our proposed pricing policy is that tenant \(i\) be charged

\[
p_i^4 = \alpha \mu_i K + \beta X(k^*) \frac{G(\mu_i, S_i)}{\sum_j G(\mu_j, S_j)}.
\]

Essentially, we propose that each tenant be charged based on its mean demand and its weighted contribution to the cloud’s peak, where the weight is a function of all tenants’ expected peaks as expressed above.

Note that it is possible through negatively correlated demand that \(S_i^2 < 0\), and this was observed in our real-world datasets, cf. Figures 5 and 9 in subsequent sections. In this case, a tenant may expect a discount for reducing coincident-peak power costs. So, in the above pricing formula we define

\[
S_i = -\sqrt{-S_i^2} \text{ when } S_i^2 < 0.
\]

**B. Empirical Evaluation**

In this section, we evaluate the alternative pricing design using real-world workload traces. A part of our empirical study is based on synthetic “sub-tenants” whose demands are created based on a real tenant but with special statistical behaviors to more clearly illustrate certain benefits of alternate pricing.

![](image.png)

Fig. 3: Individual tenants’ power demands.
Fig. 4: Our cloud’s overall power demand (summation of the tenants’ demands).

Fig. 5: $\mu_i + 2S_i$ v.s. $x_i(k^*)$ from eight tenants over 61 days. Statistics are obtained on a daily basis.

1) Workload Datasets: We use a set of eight tenant workload traces chosen from a production data center operated by IBM for its enterprise-scale customers. The datasets contain CPU utilization time-series for the servers allocated to these tenants. Each time-series spans 61 days and reports the average utilization over successive 15 minute intervals for a total of 96 samples per day. We convert these CPU utilization traces into power demands by $x_i(t) = PUE \cdot n_i(t)x^{dyn} f_i(t)$ where $n_i(t)$, $f_i(t)$ and $x^{dyn}$ are the number of servers, average CPU utilization of tenant $i$ at time $t$ and dynamic power, respectively. The power usage effectiveness ($PUE$) of a data center is the ratio of the total power delivered to it and the power used by its IT equipment. Based on measurements from these data centers, we choose the dynamic power $x^{dyn} = 127.7 W$, and $PUE = 1.8$. We ignore idle power in this study, but discuss possible extensions involving it as future work in Section VI. We show the dynamic power consumed by each tenant in Figure 3. We take a billing period to be a single day implying 61 successive billing periods for our dataset. Although billing periods are longer in practice (e.g., a month [2]), we choose this shorter billing period to have a large number of periods in our experiments.

We first make the following observations about individual tenants’ demands: (i) some of the tenants’ demands
exhibit strong daily/weekly patterns, (ii) some of the tenants’ demands have high fluctuations and might contribute much to the cloud’s peak demand, (iii) most of the tenants’ local peaks do not coincide, and (iv) some tenants’ demands exhibit significant changes (e.g., tenants 3 and 6 whose demands changes abruptly after the 40-th day and the 54-th day, respectively). We also observe that their aggregate demand exhibits non-stationary behavior and large variations as seen in Figure 4.

2) Experimental Set-up: At the end of each day (billing period) $d$, each tenant $i$ assesses its “true” mean demand $\mu_i(d)$ and demand variation $\sigma_i^2(d)$ over $d$, and the revenue-neutral cloud divides among the tenants its total cost as stated in (1) where $M(d)$ can be defined as: $M_d = \sum_i \mu_i(d)$, $K = 96$, $X(k) = \sum_i x_i(k)$ is the total tenant demand at time $k$ of day $d$, and $k^*(d)$ is the time of the coincident (aggregate) peak demand of day $d$. We choose $\alpha$ and $\beta$ according to Duke Energy tariff [2]: $\alpha = 0.02848$/kWh as energy price and $\beta = 16.193$/kW as the peak power price.

To evaluate the alternative pricing, we choose $G(\mu_i, S_i) = \mu_i + 2S_i$ for our pricing formula. In spite of many possible choices of the proxy for the cloud’s peak such as, Weibull or Gumbel distribution, $\mu_i + 2S_i$ is used as a proxy for tenant $i$’s contribution to the cloud’s peak for the following several reasons. First, we notice upon plotting $\mu_i + 2S_i$ vs. $x_i(k^*)$ for IBM real tenants, in Figure 5, that most of the sample points are scattered closely around the line $x_i(k^*) = \mu_i + 2S_i$, where $\mu_i$ was observed to be the larger component of $\mu_i + 2S_i$. Second, we find that although the tenants’ contribution to the cloud’s (tenant-aggregate) peak could be high in certain billing cycles, their
contribution to peak costs in the statistical sense may be relatively low (e.g., tenant 1), even negatively correlated (e.g., tenants 5 and 6). Third, we expect a tenant to have greater confidence in estimating $\mu_i + 2S_i$ than $x_i(k^*)$, which is the case e.g. for tenant 1.

3) **Discussion of Performance Expectations:** To evaluate the alternative pricing, we compare it with our baseline $p_i^3$. We expect the tenants‘ costs under alternative pricing to have lower fluctuations initially, due to estimating tenant $i$‘s contribution to the next day’s peak based on the previous day’s $\mu_i + 2S_i$ than by $x_i(k^*)$ (the former estimated with greater confidence). To see this, we define $\phi_i(d) = \sum_{t=1}^{d} p_i(t)/d$ as the cumulative average cost of tenant $i$ after the $d$-th billing cycle. We expect $\phi_i(d)$ to have lower fluctuations with alternative pricing than with the baseline CPP.

4) **Cyclo-Stationary Workloads with Synthetic Sub-Tenants:** In this section, we consider the demand profile of tenant 1 in Figure 3 as the aggregate demand of a hypothetical data center. This demand is modeled using estimated $\mu$ and $\sigma$ of the tenant over its approximate stationary cycles, which is a week. This deterministic behavior of the tenant over time-of-day and time-of-week can be observed even visually in the same figure. Using this model we break down the aggregate demand into synthetic demand time-series for three sub-tenants of tenant 1’s real-world trace. This process is explained in detail in Appendix VII-A. We numerically compare the performance of baseline vs. alternative pricing defined in Sections II and III-A.

Figure 6 depicts the cumulative average price over time for three statistical cases. The left column (case (a)) of Figure 6 is for the case of uncorrelated sub-tenants with identical workload statistics, i.e., $\forall i, j, \mu_i = \mu_j$ and $\sigma_i = \sigma_j$. Figure 6(a) is under baseline pricing and Figure 6(a′), is under alternative pricing, as with the other two cases. For all plots: The curves of statistically identical tenants mostly overlap. It can be seen $\phi_i(t)$ under baseline pricing has higher variation especially in the initial billing periods. Moreover, the steady state is more quickly achieved by alternative approach. This is consistent with the greater statistical confidence of the quantities used in alternative pricing. Again, tenants desire lower pricing fluctuations at all time-scales (including between billing periods), and baseline pricing can result in perceived “unfairness” in the short term, particularly by fleeting customers.

In case (b), sub-tenants have the same average demand, sub-tenant 1 has a higher variance than sub-tenants 2 and 3, but sub-tenants 2 and 3 are positively correlated and statistically identical such that: $\sigma_1 > \sigma_2, \sigma_3$ but $S_1 < S_2, S_3$. So, one expects that the positively correlated sub-tenants are charged more under alternative pricing, which is the case considering the greater gap in steady-state.

In case (c), the sub-tenants have the same average demand, sub-tenants 2 and 3 are negatively correlated, sub-tenant 1 is again uncorrelated, and $\sigma_1 < \sigma_2, \sigma_3$ but $S_1 > S_2, S_3$. One sample path of this scenario can be observed in Figure 7 with time-intervals contributing to negative correlation highlighted. As stated before, if two tenants are collaborating (even unintentionally) in reducing overall incident peak (through negative demand correlation), they should be credited through a smaller peak-component cost, which is seen in Figure 6 (c) and (c′): the sub-tenants with negative correlation are charged less under alternative pricing.

**Key insights:** (i) alternative pricing results in less variation in customers’ cumulative average cost, (ii) the tenants
reach their steady-state costs faster, (iii) alternative pricing appropriately incentivize tenants behavior, including discounts for negative demand correlation.

![Diagram](image)

Fig. 7: One sample path for 12 hours (48 samples). Case (c). Two time slots with visible contribution to negative correlation are highlighted.

![Diagram](image)

Fig. 8: $\phi_i(d)$ over 61 days (w/o tenants’ DR. “bl”: Baseline. See the full version in Appendix VII-B.

5) Non-stationary Tenant Workloads: As in Section III-B4, we show $\phi_i(d)$ of each tenant under baseline vs. alternative pricing schemes over 61 days in Figure 8 here for the different recorded tenant workloads in full. For most of the tenants, we observe more fluctuations under baseline than under alternative pricing, which is consistent with our findings in Section III-B4. This observation is also verified by calculating the coefficient of variation over the first 20 days w.r.t. tenants’ costs on the 40-th day ($p_i(40)$) for each tenant\(^2\), which is defined as

\[
cv_i = \frac{1}{19} \sum_{t=1}^{20} (p_i(t) - p_i(40))^2 / p_i(40).
\]

Recall that $p_i^3 = \alpha \mu_i K + \beta x_i(k^*)$ for the baseline and we use $(\mu_i + 2S_i)$ to approximate $x_i(k^*)$. Statistically we expect that $(\mu_i + 2S_i)$ be a good approximate and has less fluctuations than the $x_i(k^*)$ over time, which results in less variation under the alternative than under the baseline.

We do observe one exception: tenant 5 has more fluctuation under alternative than under baseline. This is due to the fact that $S_5^2$ is almost always negative for the first 15 days (as shown in Figure 9) and $x_5(k^*)$ is always positive.

\(^2\)After 40 days, tenant 3’s demand changes abruptly.
and does not vary much (uniquely for tenant 5). Consequently, $\mu_5 - 2\sqrt{-S_5^2}$ is not as good an approximation for $x_5(k^*)$. However, consistent with the desirable properties of alternative pricing mentioned in Section II: Tenant 5’s demand is negatively correlated with others, which helps reduce cloud’s aggregate peak demand, and samples $x_5(k^*)$ or $\mu_5 + 2\sigma_5$ do not capture such desirable negative-correlation behavior. Figure 10 shows a comparison of some tenants’ costs (tenants 1, 3, and 5) for the two different pricing schemes. We observe that tenant 1 is charged much less under the alternative than under baseline, which is because some tenants (e.g., tenant 3) might have contributed more to the cloud’s peak power costs by having positive demand correlations with others, although their instantaneous share of the cloud’s peak power might be relatively low. Note, tenant 5 under alternative pricing has negative costs (possibly rewards) by having negative contribution to the cloud’s peak power. We also observe that after the 40-th day, tenant 3’s demand changes abruptly, and so does its cost. However, tenant 3’s costs under alternative pricing after the 40-th has much less variation than under baseline pricing (again, this is desirable). Also, as shown in Figure 9, $S_5^2 < 0$ for the first 15 days, which results in greater cost discounts for tenant 5 under
alternative compared to baseline pricing, as described above.

**Key insights:** (i) the cumulative average costs under alternative pricing have lower variation, and (ii) alternative pricing can help the cloud motivate desirable tenant behavior by discounting tenants’ with negative demand correlations.

### IV. Cost Attribution When Tenants Engage in Demand Response

We now consider the effects of tenant workload control/acutation via perfect demand shedding\(^3\). By supposing that each tenant handles a large and diverse job arrival process, demand shedding can be accomplished by choosing a certain proportion of incident jobs at random (in unbiased fashion) to reject, resulting in the same proportionate reduction in overall workload and power consumption of the tenant on average over a long period of time. Alternatively, a workload taxonomy can be developed and employed to classify incident jobs (and their expected workloads) and decide which ones to shed.

In this section, we first prove under simplifying assumptions\(^4\) that at Nash equilibrium, a tenant whose demand has greater positive correlation with the aggregate other tenants will experience greater charges under the alternative pricing strategy. Subsequently, under more realistic setting using the real-world workload traces described above, we numerically explore the effects of demand response.

#### A. A Game-Theoretic Analysis

1) **Simplifying Assumptions:** To develop a game-theoretic model, suppose tenant \(i\) will optimize net utility

\[
v_i(\mu) = u_i(\mu_i) - p_i(\mu)\tag{7}
\]

over their mean demand \(\mu_i\); where: \(p_i\) is the “alternative” pricing scheme (5) with the aggregate peak \(X(k^*)\) replaced by the “statistical aggregate peak” \(M + 2S\); \(\mu\) is a vector of all tenants’ mean demands, and the utility \(u_i\) is continuous, increasing, concave and bounded. Note that Brouwer’s theorem gives existence of a Nash equilibrium [18]. We make the following assumptions to simplify our analysis in this subsection:

- Observation: The tenants have perfect information regarding the statistics of their demand in the next billing cycle; in particular, the second-order statistics are used to predict contribution to coincident peak-power consumption on which the utility bases its electricity charges to the data center\(^5\). In the numerical experiments described in the following, second-order correlations among tenant demands were assumed estimated by the cloud based on previous demand activity; in practice, these may be noisy.

---

\(^3\)In future work, we will consider imperfect load shedding and demand response by DVFS or load deferral as well, the latter used to desynchronize tenant demands and reduce coincident peak-power consumption. Also, techniques of workload migration, leading to workload consolidation and server shut-down, have been proposed to reduce idle power in a data-center [16], [17], objectives not addressed in this paper.

\(^4\)In particular that demand shedding does not affect demand co-variances.

\(^5\)Each tenant could instead formulate direct estimators of contribution to peak workload in the next billing cycle \(t, \tilde{x}_i(k^*(t))\), based on these quantities from past billing cycles, \(x_i(k^*(s))\) for \(s < t\), but the statistical confidence associated of such estimators is much lower than those for \((\mu_i + 2S)\) in non-stationary settings, and even in stationary ones because estimates are informed by far less data.
• Control: We also assume shedding load does not affect the second-order statistics (demand variation) of the tenants, and that tenant-demand cross-correlations are non-negative.

Again, in the following numerical study we relax these assumptions. Given the above assumption on control, we intend to prove that the Nash equilibrium prices increase with demand variation and (positive) demand correlation.

2) When Demands Are Uncorrelated: We first consider demands that are uncorrelated (\( \forall i \neq j, c_{i,j} = 0 \)). Given this, our pricing policy in (7) is taken as

\[
p_i(\mu) = \alpha \mu_i K + \beta (M + 2S) \frac{\mu_i + 2\sigma_i}{M + 2 \sum_j \sigma_j}
\]

where we note that

\[
0 < \varepsilon := 2 \left( \sum_i \sigma_i - S \right) := 2 \left( \sum_i \sigma_i - \sqrt{\sum_i \sigma_i^2} \right).
\]

(8)

The following two claims present the conditions for Nash equilibrium and the tenants’ demands at Nash equilibrium.

**Claim 1.** If \( C \) is diagonal, all utilities \( u_i(\mu_i) \) are concave in \( \mu_i \), and

\[
\forall i, \sum_{j \neq i} (\mu_j + 2\sigma_j) > \mu_i + 2\sigma_i,
\]

then there exists a unique Nash equilibrium to which “continuous best-response” (i.e., better response) dynamics,

\[
\forall i, \dot{\mu}_i = \gamma_i \frac{\partial v_i}{\partial \mu_i},
\]

(10)

converge for any positive parameters \( \gamma_i > 0 \).

**Proof:** Rosen’s conditions [19] for uniqueness of the Nash equilibrium and convergence to it by continuous best-response is that the symmetric matrix with \( (i,j) \) entry

\[
\frac{\partial^2}{\partial \mu_i \partial \mu_j} (\gamma_i v_i + \gamma_j v_j)
\]

is everywhere strictly negative definite. One can directly show that \( \forall i, j \),

\[
\frac{\partial^2 v_i}{\partial \mu_i \partial \mu_j} < 0,
\]

where the claim’s hypothesis implies, \( \forall i \neq k, \partial^2 p_i / \partial \mu_i \partial \mu_k > 0 \).

Note that (9) requires that the number of tenants \( N > 2 \). Also, (9) needs to hold for all \( \mu \) in a neighborhood of the unique Nash equilibrium \( \mu^* \), which we’ll see from the following result is such that \( \mu_i^* + 2\sigma_i = c \) for a positive constant \( c \), i.e., (9) will indeed hold when \( N > 2 \).
Let
\[ c := \frac{(N-1)\beta}{N^2(-a+\alpha K + \beta)} \]
and
\[ z := \frac{\beta}{N^2} \cdot \frac{Na - \alpha K - \beta}{-a + \alpha K + \beta}. \]

The following claims how tenant charges at Nash equilibrium, \( p^* \), (fairly) increase with tenant demand variation, \( \sigma^2 \).

**Claim 2.** If \( C \) is diagonal,
\[ \exists a > 0 \text{ s.t. } \forall i, \ u_i(\mu_i) = a\mu_i, \tag{11} \]
\( i.e., \) linear utilities with common parameter \( a \),
\[ 2 \max_i \sigma_i \leq c \tag{12} \]
and
\[ \forall i, \ (1 - \sigma_i S^{-1}) z > \alpha K, \tag{13} \]
then the prices at Nash equilibrium \( \mu_i^* = c - 2\sigma_i \) satisfy
\[ \forall i, \ \frac{\partial p_i^*}{\partial \sigma_i} > 0. \]

**Proof:** The solution of the first-order necessary conditions for the (unique) Nash equilibrium, \( \forall i, \)
\[ 0 = \frac{\partial v_i}{\partial \mu_i} \]
\[ = a - \alpha K - \beta + \beta \varepsilon \frac{\sum_{j \neq i}(\mu_j + 2\sigma_j)}{(\sum_j(\mu_j + 2\sigma_j))^2} \]
\[ =: a - \alpha K - \beta + \beta \varepsilon \frac{-\mu_i - 2\sigma_i + y}{y^2}. \]

Thus, at Nash equilibrium, \( \mu_i + 2\sigma_i \) is a constant over tenant-index \( i \) satisfying
\[ (\mu_i + 2\sigma_i) \beta \varepsilon =: (a - \alpha K - \beta) y^2 + \beta \varepsilon y \]
\[ \Rightarrow y \beta \varepsilon = N(a - \alpha K - \beta) y^2 + N \beta \varepsilon y \]
\[ \Rightarrow y = \frac{(N-1)\beta \varepsilon}{N(-a + \alpha K + \beta)} =: Nc \]
So, the Nash equilibrium is given by,
\[ \forall i, \ \mu_i^* = c - 2\sigma_i \geq 0, \]
recalling that \( \sigma_i \) is assumed fixed in this section, where non-negativity is by assumption (12). Thus, \( \forall i, \) the price at Nash equilibrium for tenant \( i \) is
\[ p_i^* := \alpha K(c - 2\sigma_i) + \beta(M + 2S) \frac{c}{Nc} \]
\[ := \alpha K(c - 2\sigma_i) + \beta(Nc - \varepsilon) \frac{c}{Nc} \]
\[ = ((\alpha K + \beta)\varepsilon^{-1} - \beta N^{-1})\varepsilon - 2\alpha K\sigma_i \]
\[ =: z\varepsilon - 2\alpha K\sigma_i \]
\[ \Rightarrow \frac{\partial p_i^*}{\partial \sigma_i} = z2(1 - \sigma_i S^{-1}) - 2\alpha K. \]

Note that according to the proof, \( \mu_i + 2\sigma_i = (a - \alpha K - \beta)\frac{y^2}{(\beta \varepsilon)} + y \) at Nash equilibrium for all tenants \( i \), i.e., each tenant’s \( \mu_i + 2S_i \) will be the same.

3) When Tenant Demands Are Correlated: More generally for correlated tenant demands (i.e., \( C \) is not diagonal), consider the pricing policy (5)

\[ p_i(\mu) = \alpha \mu_i K + \beta(M + 2S) \frac{\mu_i + 2S_i}{M + 2\sum_j S_j}. \]  

(14)

In this section, we assume \( S_i^2 \geq 0 \) for all \( i \) (again, in the numerical section we show are instances of negatively correlated tenants).

We now provide conditions for the existence of Nash equilibrium in corollary 1 and then show that the Nash equilibrium prices increase in both their demand standard deviation \( \sigma_i \) as well as their demand cross-correlation \( \sqrt{S_i^2 - \sigma_i^2} \), in corollary 2 and 3, respectively.

By the same argument for Claim 1, we get

**Corollary 1.** If all utilities \( u_i \) are concave,

\[ \forall i, \sum_{j \neq i}(\mu_j + 2S_j) > \mu_i + 2S_i, \]

and

\[ \sum_i S_i > S, \]  

(15)

then there exists a unique Nash equilibrium to which “continuous best-response” (i.e., better response) dynamics,

\[ \forall i, \quad \dot{\mu_i} = \gamma_i \partial u_i / \partial \mu_i, \]  

(16)

converge for any positive parameters \( \gamma_i > 0 \).

**Lemma 1.**

\[ S^2 = \sum_i \sigma_i^2 + \frac{1}{2} \sum_i (S_i^2 - \sigma_i^2) \]

**Proof:** Note that in \( \sum_i S_i^2 \), the terms \( c_{i,j} \), for \( i \neq j \), appear \( N - 2 \) times, while the diagonal terms \( c_{i,i} = \sigma_i^2 \)
appear $N - 1$ times. Thus,

\[ (\sum_i S_i)^2 = \sum_i S_i^2 + 2 \sum_{j<i} S_j S_i \]

\[ = \sum_i (S_i^2 - S_{i-1}^2) + 2 \sum_{j<i} S_j S_i \]

\[ = NS^2 - \sum_i S_{i-1}^2 + 2 \sum_{j<i} S_j S_i \]

\[ = NS^2 - ((N-2)S^2 + \sum_i \sigma_i^2) + 2 \sum_{j<i} S_j S_i \]

\[ = S^2 + (S^2 - \sum_i \sigma_i^2) + 2 \sum_{j<i} S_j S_i \]

\[ = 2S^2 - \sum_i \sigma_i^2 + 2 \sum_{j<i} S_j S_i \]

\[ \Rightarrow S^2 = \frac{1}{2} \left( \sum_i S_i^2 + \sum_i \sigma_i^2 \right) \]

Again, note that for the case of uncorrelated demands, $S_i = \sigma_i$ and $S^2 = \sum_i \sigma_i^2$. By (17), (15) holds when tenant demands are only positively correlated, i.e., $\forall i, j, c_{i,j} \geq 0$. Because $\partial S_j^2 / \partial \sigma_j^2 = 1$, Claim 2 also generalizes by using this lemma.

**Corollary 2.** If (15), utilities are linear with common slope $a > 0$,

\[
2 \max_i S_i \leq c
\]

and

\[
\forall i, \quad (1 - S_i S^{-1})z > 2\alpha K \quad (17)
\]

then the prices at Nash-equilibrium satisfy,

\[
\forall i, \quad \frac{\partial p^*_i}{\partial \sigma_i} > 0.
\]

**Corollary 3.** If (15), utilities are linear with common slope $a > 0$,

\[
2 \max_i S_i \leq c,
\]

\[
\forall i, \quad (1 - \frac{1}{2} S_i S^{-1})z > 2\alpha K, \quad (18)
\]

and

\[
\forall i, \quad S_i^2 > \sigma_i^2.
\]
then the prices at Nash-equilibrium satisfy,

$$\forall i, \frac{\partial p^*_i}{\partial \sqrt{S^2_i - \sigma^2_i}} > 0.$$ 

In summary, Corollary 3 gives conditions under which the Nash equilibrium prices increase with the degree to which tenant $i$’s demand correlates with the other tenants $\sqrt{S^2_i - \sigma^2_i}$ to create coincident peaks in demand.

B. Evaluation with Tenants’ Demand Response

1) Workload Forecasting Overview: In this section we will use demand response for tenants. Causal DR will require accurate workload forecasting. Forecasting short-term (e.g., day ahead) behavior for similar such datasets has an extensive literature. For example, deterministic cyclic/seasonal components can be assessed (based on prior observations) and removed from the raw data, leaving what may be a stationary residual. Given that, one can employ standard time-invariant ARMA-type estimators [20] whose meta-parameters (particularly lags) can also be determined by optimizing over prior observations. However, it’s often unclear how much “detrending” is required (e.g., which moments) and the residual may ultimately have time-varying statistics (i.e. is non-stationary) requiring ARMA estimators with time-varying/adapted coefficients and lags. In these cases, techniques of adaptive filters such as LMS and RLS, and associated heuristics, are commonly used [21], often with short lags. In this paper, we use first-order auto-regressive estimators, or even more simply take the previous days’ data as a proxy for the next (i.e., a kind of zeroth-order estimator), and leave more sophisticated workload forecasting in this context for future work.

2) Tenants’ DR under Alternative Pricing: We assume that the cloud estimates the inter-tenant demand correlations $S^2_i(d)$ over the previous day, and communicates these estimates only to the corresponding tenants. At the end of day $d$ in an alternative scenario, tenant $i$ is charged,

$$p^4_i = \alpha \mu_i K + \beta X(k^*(d)) \frac{\mu_i(d) + 2S_i(d)}{\sum_j(\mu_j(d) + 2S_j(d))},$$

again recalling we set $S_i = -\sqrt{-S^2_i}$ when $S^2_i < 0$.

For purposes of determining at the start of day $d$ the fraction $1 - \lambda_i(d)$ of its demand it will shed ($\lambda_i(d)$ is the control variable), we assume tenant $i$ will rely on a simple AR(1) estimate of its incident demand mean,

$$\hat{\mu}_i(d) = 0.5\hat{\mu}_i(d-1) + 0.5\mu_i(d-1)/\lambda_i(d-1),^6$$

taking $\mu_i(0) = 0 = \hat{\mu}_i(0)$ and $\lambda_i(0) = 1$. The same model is used for estimating tenant’s incident demand variation,

$$\hat{\sigma}^2_i(d) = 0.5\hat{\sigma}^2_i(d-1) + 0.5\sigma_i^2(d-1)/\lambda_i^2(d-1),$$

taking $\sigma_i(0) = 0 = \hat{\sigma}_i(0)$. We also assume that each tenant will forecast inter-tenant demand correlations for day

^6 Generally take $\frac{0}{0} = 0$
Fig. 11: $\phi_i(d)$ and $\lambda_i(d)$ (t) after tenants’ DR, 60 days, 100 sample paths, thicker lines show the average and thinner-dotted lines show the confidence bars: (a) all sub-tenants statistically identical and uncorrelated, (b) only sub-tenants 2 and 3 are statistically identical and positively correlated, (c) only sub-tenants 2 and 3 are statistically identical and negatively correlated.

$d$ as that computed by the cloud for day $d - 1^7$, also considering the effect of demand-shedding and assuming that other tenants will not change their shedding strategy$^8$. More specifically, tenant $i$ takes as its causal estimate of $S_i^2(d)$:

$$\hat{S}_i^2(d) = (S_i^2(d - 1) - \sigma_i^2(d - 1)) \frac{\lambda_i(d)}{\lambda_i(d - 1)} + \lambda_i^2(d) \hat{\sigma}_i^2(d).$$

Again, to determine its load shedding for day $d$ at the start of day $d$, tenant $i$ will optimize its net utility over $\lambda_i(d)$. Since the tenant does not have the information of $X(k^*(d))$ at the beginning of day $d$, it may simply take $X(k^*(d)) \approx \sum_j (\mu_j(d) + 2S_j(d))$. Therefore, the net utility of tenant $i$ can be assumed of the form

$$a_i \log(b_i \hat{\mu}_i(d) \lambda_i(d) + 1) - (\alpha K \hat{\mu}_i(d) + \beta \hat{\mu}_i(d)) \lambda_i(d) - \beta 2 \hat{S}_i(d).$$

3) Discussion of Performance Expectations: For our experiments, we used the same utility coefficients$^9$, i.e., $\forall i, j, a_i = a_j = a, b_i = b_j = b$. The correlation between tenants will still exist after DR. Therefore by the results of Section IV-A, we expect to see prices increase with tenant correlations $S_i$. Moreover, we expect demand shedding by tenant $i$ will have amplified effect by reducing $S_i$ when $S_i > 0$.

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7Alternatively, the tenants could use even earlier days’ estimates for this purpose.
8As consideration of only unilateral defection of a collective play action in the definition of a Nash equilibrium of a non-cooperative game. Indeed, were the tenant demands stationary, such daily play-actions could eventually lead to a Nash equilibrium, cf. Section IV-B4.
9The effects of differences among tenant utility-parameters are straightforward, see Appendix VII-A1.
4) Cyclo-Stationary Workloads of Synthetic Sub-Tenants: In this section we look at effects of demand response under alternative pricing for three synthetic sub-tenants. Recall from Section III-B4 that these sub-tenants are generated for three different cases each using time-of-day and time-of-week detrending of $\mu$ and $\sigma$ derived from a real-world trace (of tenant 1 in Figure 3), as explained in Appendix VII-A.

For case (a), with uncorrelated and statistically alike tenants, the cumulative average costs ($\phi_i$) and fraction of admitted demand ($\lambda_i$) under demand response for alternative pricing is given in Figure 11 (with confidence-interval outlines). The plots overlap as the tenants are statistically identical. Compared to the case without demand response in Section III-A, steady-state is reached in greater time owing to the action of demand response. The sub-tenants shed on average 30% of their demand, resulting in lower prices over time. The oscillation of $\lambda$ are due to weekly pattern of raw demand used to generate the synthetic sub-tenants’ demand.

For case (b), statistically identical sub-tenants 2 and 3 have positive correlation while sub-tenant 1 is uncorrelated with lower $\mu_1 + 2S_1 = \mu_1 + 2\sigma_1$. Sub-tenants 2 and 3 contribute more to aggregate demand variation ($M + 2S$) and so alternative pricing charges them more than sub-tenant 1 than without demand response. However, sub-tenants 2 and 3 shed less demand than sub-tenant 1, because demand-shedding will be amplified through reducing (positive) correlation between their demands.

For case (c) in which sub-tenants 2 and 3 have negative correlation, sub-tenants 2 and 3 are charged less than without demand response. Additionally sub-tenants 2 and 3 shed more because demand shedding increases their (negative) correlation. We also observe that demand response affects $S_i^2$ roughly proportionate to $\lambda_i$, so the financial benefit of demand shedding in this case will depend on the relative size of mean-power and peak-power costs.

**Key insights:** (i) Overall $S_i^2$ values as main contributors to the peak-component charges are decreased considerably after demand response. (ii) Correlated tenants’ shedding has a significant effect on their correlations.

5) Non-stationary Tenant Workloads: The numerical results for the cyclo-stationary workloads of sub-tenants of tenant 1 under DR with alternative pricing are similar to those of the full (unmodified) non-stationary tenants shown
First note that tenant 1’s control actions have less fluctuation than tenant 2, which can be explained by the fact that tenant 1’s incident workload statistics ($\mu_1, \mu_1 + 2S_1$) have less variance. Tenant 3 begins to shed load after its demand ramps up on the 40-th day, whereas other tenants’ demands are not shedded at all. Also note that in Figure 12(b), the admitted demands of tenant 1 and tenant 2 after load shedding become more similar than their incident demands; similarly for tenant 3 after the 40-th day, in part owing to the assumption of the same net-utility/revenue parameters ($a_i, b_i$) and that the proportion of incident mean demand $\mu_i$ to $S_i$ is roughly the same for these tenants (and this proportion does not change significantly with demand shedding). Moreover, although tenant 1 has much larger mean than tenant 2 (as shown in Figure 3), it has to shed much of its demand to maximize profit due to the log form utility function and diminishing marginal returns of increasing mean demand. Thus, these effects of DR are intuitive.

![Cumulative average costs with tenants DR. "*": Game with synthetic tenant 1, real tenants 2,3 and 5.](image)

In a comparative study, we create a synthetic tenant 1 with greater demand variation as follows: We estimate the (short-term moving) average $m_1(t)$ of $x_1(t)$ (the real tenant 1’s demand), and define the synthetic tenant 1’s demand as $\tilde{x}_1(t) := m_1(t) + \theta(x_1(t) - m_1(t))$ for $\theta = 4$. We conduct two sets of DR experiments: one with real tenants 1,2,3,5 and the other with synthetic tenant 1 and real tenants 2,3,5. The cumulative average costs of each tenant in both experiments are shown in Figure 13. We find that synthetic tenant 1 has much higher costs than real tenant 1 since the former’s demand has much higher incident variance (and higher $c_{1,j}$ henceforth if $c_{1,j} > 0$) which doesn’t change under DR. This is consistent with 17 and Corollary 2 (obtained at Nash equilibrium in far more idealized settings). Furthermore, we observe that the other (real) tenants suffer from higher costs in the presence of synthetic tenant 1, with the exception of tenant 5. This is because the positive demand cross-correlations increase when we increase tenant 1’s demand variance. In other words, if a tenant is willing to reduce its demand variation (and cross-correlation) by DR, other tenants whose demands are positively correlated can benefit from it and have

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10We choose real tenant 5 since it has little correlation or sometimes even negative correlation with tenant 1.
lower costs. Since tenant 5 has little positive correlation and sometimes negative correlation with tenant 1, its costs are not affected as much.

We observe that compared to Figure 5, \( \mu_i + 2\sigma_i \) is more aligned with \( x_i(k^*) \) after tenants’ DR (as shown in Figure 15 in Appendix VII-B). Tenants who have larger \( \mu_i + 2\sigma_i \) before DR (e.g., tenant 1 in Fig. 5) shed more demands than others, which is also consistent with our observations from Fig. 12(b).

**Key insights with DR:** (i) Tenants’ load shedding exhibits lower fluctuation when the workloads have less variance. (ii) Tenants are likely to have the same/similar mean demand (but with different variance possibly) when they have the same parameters for utility functions. (iii) Even with load shedding, tenants with higher variance will have higher costs. (iv) Tenants with positively correlated demands can benefit from each other’s load shedding.

V. **Related Work**

**Pricing design in clouds.** There is a growing body of related work on cloud pricing, and various techniques such as dynamic pricing [22], auction-based pricing [23], Nash bargaining [24] with either single or multiple strategic cloud providers [25], [26], have been proposed. However, our focus on the impact of electric utility tariff structure (especially the impact of peak-based pricing) on the cloud’s pricing strategies (and the tenants’ DR in terms of power thereafter) is novel. There is recent work on cloud’s DR and related pricing design, e.g., in [27] a reverse-auction framework is proposed for co-location data centers to incentive tenant DR in order to maximize the cloud’s profit. Similarly, in [28] a prediction-based pricing scheme is proposed for the electric utility to encourage cloud’s load shedding to meet a peak-related target. Again, the key distinguishing aspect of our work is its focus on notions of fairness. The pricing schemes developed in all these works are likely to suffer from the unfairness problems and oscillatory costs for tenants that we identified for our baseline. **Data center power management.** Data center power management (including DR) has been explored extensively and we only cite a few representative examples. We find it useful to classify DR-related work along two dimensions. First, a variety of power/resource control techniques have been explored. One may view these as being (often implicitly) based on demand shedding (e.g., admission control, equipment slow/shutdown, quality-of-service reduction) [29], [16], [30], [31], demand delaying (e.g., scheduling) [32], demand transfer (e.g., migration) [33], demand modulation using batteries [4], [34], or combinations of these. Second, DR algorithms have been developed for a variety of pricing schemes including coincident peak pricing [15], peak-based pricing [35], and many forms of real-time pricing. Although we have only considered a simplified form of load shedding as our DR, we take other control knobs as possible extensions to our work. Finally, two detailed surveys cover many of these issues and serve as excellent resources for understanding this area [5], [6].

VI. **Directions for Future Work**

In future work, we will attempt to extend our analytical results to consider effects on second-order statistics of demand shedding and to consider noisy observations and play-actions (the latter informed by more sophisticated workload forecasting for the next billing cycle). Also, using our real-world datasets, we will consider other types of demand response, e.g., DVFS or demand deferral via scheduling, possibly in combination with demand shedding.
Moreover, we will explore the performance benefits of techniques of workload migration, leading to workload consolidation and server shut-down, which have been proposed to reduce idle power in a data center. Finally, we are pursuing more detailed workload classification that indicates workload of specific arriving jobs; with such data, we may be able to to formulate a job taxonomy and classification system to predict sensitivity to deferral\textsuperscript{11} and the demand footprint of specific jobs, and thus improve the accuracy of workload shedding in particular.

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\textsuperscript{11}Excessive job deferral may amount to dropping.


VII. APPENDIX

A. Generating synthetic component tenants

In this appendix, we describe how the sub-tenants demands are generated. First, let \( x(t) \) be the raw demand of tenant 1 of Figure 3. The time interval of the trace was approximately 8 weeks (with a sample every 15 minutes), so that there are 8 samples taken at the same time of the same day of week. For each sample \( t \) during this 8-week period, let \( \delta(t) \) be the day-of-week and \( \tau(t) \) be the time-of-day. A pattern was observed for this data over each week which can be easily observed in Figure 3. Therefore, we removed this pattern by removing periodic time-of-day and time-of-week effect \( \bar{x}(\delta(t), \tau(t)) \) from the raw trace \( x(t) \). The obtained residual after detrending is as follows:

\[
r(t) = x(\delta(t), \tau(t)) - \bar{x}(\delta(t), \tau(t)),
\]

where

\[
\bar{x}(\delta(t), \tau(t)) = \frac{1}{8} \sum_{s: \delta(s)=\delta(t) \text{ and } \tau(s)=\tau(t)} x(s).
\]

In Figure 14, we show the autocorrelation function of (mean zero) residual \( r \), which is approximately white as we subsequently assume for simplicity\(^{12} \). Given this assumption, we can compute the sample variance of the residual \( r \) at each time-of-day,

\[
\sigma^2(\delta(t), \tau(t)) = \frac{1}{8} \sum_{s: \delta(s)=\delta(t) \text{ and } \tau(s)=\tau(t)} r(s)^2.
\]

![Sample Autocorrelation Function (ACF)](sample_acf.png)

Fig. 14: Autocorrelation of detrended tenant demand.

By using this model which is based on first and second order statistics of the raw data, we can model synthetic sub-tenant data. Therefore, \( \bar{x}(\delta(t), \tau(t)) \) and \( \sigma^2(\delta(t), \tau(t)) \) are broken down to desired number of tenants by fixed

\(^{12}\) A more complex representation of \( r \) could be based on a FIR approximation of its inverse whitening filter (driven by actual white noise) [20].
weight matrices $\mathbf{A}$ and $\mathbf{D}$ respectively over the whole period of simulation. The synthetic sub-tenant mean weight $\mathbf{A}$ and covariance matrix weight $\mathbf{D}$ is normalized so that their entries $\sum_j a_j = 1$ and $\sum_{i,j} d_{ij} = 1$. This is to achieve synthetic sub-tenants data such that their aggregate behavior is statistically similar to the raw data. It is necessary that $\sum_j a_j = 1$ holds to have the same aggregate mean and for $\mathbf{D}$, considering $\text{var}(X + Y) = \text{var}(X) + \text{cov}(X,Y) + \text{cov}(Y,X) + \text{var}(Y)$, it is desired that aggregate demand of the synthetic sub-tenant data have the same variance as the raw tenant data which is satisfied by $\sum_{i,j} d_{ij} = 1$. Consequently, for sample $t$, we took the covariance of the sub-tenants to be $\sigma^2(\delta(t), \tau(t))\mathbf{D}$. So to generate the $t^{th}$ samples $y_j(t)$ of the $j^{th}$ sub-tenant, we generated i.i.d. Gaussian $\text{N}(0,1)$ samples $w_j(t)$ and set

$$y(t) = \bar{x}(\delta(t), \tau(t))\mathbf{A} + \sigma(\delta(t), \tau(t))\mathbf{B}w(t),$$

where $\mathbf{B}$ is a lower triangular matrix of the Cholesky decomposition of $\mathbf{D}$. $\mathbf{B}$ is used to apply correlation between synthetic sub-tenant data to present specific cases for our study.

1) Discussion of experimental variations: There are possible variations for the set of experiments done in previous sections of this work. Some measurement and actuation errors could be introduced into the system. The source of this error could be noisy measurement or calculation of statistics of tenants as well as noise engaged in actuation for demand response.

The other variation is using different utility functions. In this study we are using the same coefficients for utility function $a$ and $b$. Therefore, tenants are shedding similarly. By adopting different $a$ and $b$ sub-tenants can control the amount of their shedding. It is obvious that increasing $a$ and $b$ values will increase the utility and therefore decrease the amount of shedding.

### B. Additional figures

![Fig. 15: $\mu_i + 2S_i$ v.s. $x_i(k^*)$ with tenants’ demand response.](image-url)
Fig. 16: $\phi_i(d) = \sum_{t=1}^{d} p_i(t)$ without tenants' demand response (61 days). The solid curves are obtained under alternative pricing while the curves with “*” represent the baseline pricing.

Fig. 17: $S_i^2$ for each tenant over two months without tenants’ demand response.