Distributed ALOHA game with partially rule-based cooperative, greedy, and vigilante players

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I. INTRODUCTION

Many pervasive protocols which continue to be used in distributed/decentralized communication networking today fundamentally rely on “cooperative behavior” of the participating end-hosts/users. In particular, TCP in layer 4 and MAC in layer 2 of the OSI protocol hierarchy prescribe distributed transmission back-off rules in response to detected congestion and, in the case of some MAC protocols, randomized ramp-up to desynchronize demand (again, in decentralized fashion) and prevent congestion during recovery. Such important flow and congestion control mechanisms can be undermined through the spread of malware targeting them, or deliberately by the end-users themselves employing non-cooperative protocol modifications in order to take advantage of the expected cooperative behavior of most other users, e.g., “turbo” TCP.

As a result, distributed networking games modeling every user as non-cooperative have been extensively investigated over the last decade - the idea being that this fully non-cooperative scenario is one that will lead to a fair Nash equilibrium, giving performance comparable to that of the fully cooperative case. More realistically, at least in the near term, is a hybrid model where some users abide by a “standard” cooperative protocol while others greedily depart from it. Again, in such a system, the greedy players need only fear each other as a critical mass of greedy players (perhaps even just two) can result in deadlock (excessive congestion resulting in minimal throughput among them).

In this note, we consider such a hybrid system involving at least one greedy player exploiting a plurality of cooperative players. Rather than relying on an at least somewhat centralized supervisor to detect and guard against the greedy player, one or more “vigilante” players adopt cooperative play only when they perceive that they are getting their fair share of networking resources. If, however, they deem their perceived share of networking resources as less than fair, the vigilante players behave in greedy fashion, causing deadlock with the greedy players, essentially to incentivize greedy players to return to cooperative behavior.

A. Cooperative Players

Suppose $N - 2$ players are always cooperative in the sense that their transmission probability (play) is fixed at $1/N$.

On this platform, consider a game between the remaining two players one of which is “greedy” and the other “vigilante” in the following sense. Let $p_g$, resp. $p_a$, be the transmission probability of the greedy, resp. vigilante, player. So, the (steady-state) throughput of the greedy and vigilante player is, respectively,

$$
\theta_g := \Theta(p_g, p_a) := p_g(1 - p_a)(1 - 1/N)^{N-2} \quad \text{and} \quad \theta_a := \Theta(p_a, p_g).
$$

The throughput of cooperative player $i$ is obviously,

$$
\theta_i = (1 - p_g)(1 - p_a)(1 - 1/N)^{N-3}/N \quad \forall \ a \neq i \neq g.
$$

Let $\theta_g = (\theta_g')^{-1}(M)$ which optimizes the net utility of the greedy player,

$$
v_g(p_g, p_a) := u_g(\theta_g) - M\theta_g.
$$

Let the net utility of the vigilante player be

$$
v_a(p_a, p_g) := u_a(\theta_a) - A \sum_{i \neq a}(\theta_i - \Phi)^2 - M\theta_a,
$$

where the sum of the vigilante (second) term is over all players $i$, the “vigilante parameter” $A > 0$, and, by simple substitution,

$$
\sum_{i \neq a}(\theta_i - \Phi)^2 = [\theta_g - \Phi]^2 + (N - 2)\{(1 - p_g)(1 - p_a)(1 - 1/N)^{N-3}/N - \Phi\}^2.
$$

Note that we have assumed convex utilities $u$ and throughput (goodput) based costs, $M\theta$. Also, explain how under CSMA the vigilante player can estimate the vigilante cost component.

B. Greedy player

The greedy player’s desired throughput is more than their fair share,

$$
\theta_g^* > \Phi.
$$
At the $k$th (quasi-stationary play) iteration, the greedy player employs

$$p_g^{(k)} = \begin{cases} p_g^{(k-1)} & \text{if } \theta_g^{(k-1)} \geq \theta_g^* \\ \theta_g p_g^{(k-1)}/\theta_g^{(k-1)} & \text{if } (1 - \varepsilon_g) \Phi < \theta_g^{(k-1)} \leq \theta_g^* \\ 1/N & \text{else} \end{cases}$$

where $0 < \varepsilon_g < 1$. Note that the net utility is optimized at

$$\theta_g^{*} p_g^{(k-1)}/\theta_g^{(k-1)} = \text{arg} \max_{p_g} u_g(\Theta(p_g, p_a^{(k-1)})) - M \Theta(p_g, p_a^{(k-1)}).$$

C. Vigilante player

The vigilante player’s desired throughput equals their fair share

$$\theta_a^* := (u_a^-)^{-1}(M) = \Phi \quad (4)$$

when the vigilante term is zero (i.e., when all players are playing fairly with $p = 1/N$). At the $k$th iteration, the vigilante player employs

$$p_a^{(k)} = \begin{cases} \Phi p_a^{(k-1)}/\theta_a^{(k-1)} & \text{if } \theta_a^{(k-1)} \leq (1 - \varepsilon_a) \Phi \\ 1/N & \text{else} \end{cases}$$

where $0 < \varepsilon_a < 1$.

So, the greedy player will cooperate only when s/he is getting significantly less (i.e., by factor $(1 - \varepsilon_g)$) than their fair share, and otherwise attempt to obtain $\theta_g^*$, which is presumed more than their fair share, by optimizing their net utility. On the other hand, the “vigilante” player will attempt to obtain their fair share by “greedy means” only if their current share is significantly less (i.e., by factor $(1 - \varepsilon_a)$) than their fair share.

If there is more than one vigilante player, deadlock may ensue under the above rules. To avoid deadlock, the following modification to vigilante play is used: at the $k$th iteration, a vigilante player employs

$$p_a^{(k)} = \begin{cases} \Phi p_a^{(k-1)}/\theta_a^{(k-1)} & \text{if } \theta_a^{(k-1)} \leq (1 - \varepsilon_a) \Phi \\ 1/N & \text{else} \end{cases}$$

where $T_a$ is a time interval based on the number of players, $N$.

That is, if vigilante players have departed from fair play and receive negligible throughput for $T_a$ successive slots, then they will back-off. The action of the greedy player(s) will synchronize the action of the vigilante players and so this back-off will be synchronized. Back-off synchronicity is important because if only one vigilante player backs off, it may substantially return to unfair play before the other vigilante players decide to back-off.

II. CYCLIC BEHAVIOR

Let $p = (p_a, p_g)$ be the joint play of the vigilante and greedy players. Because the mapping $p^{(k-1)} \rightarrow p^{(k)}$ is not continuous, there may not exist a Nash equilibrium point.

Beginning from fair play, we now derive conditions for the following cyclic behavior:

1) from fair share, the greedy player attempts to get more than his/her fair share,
2) so the vigilante player gets significantly less than his/her fair share,
3) so the vigilante player acts in a greedy fashion,
4) so the greedy player gets significantly less than their fair share,
5) so the greedy player cooperates,
6) so the vigilante player cooperates so that all players get their fair share,
7) go to step 1

That is, by attempting to achieve $\theta_g^* > \Phi$, the greedy player will cause $\theta_a < (1 - \varepsilon_a) \Phi$; and when both players are greedy, the result will be $\theta_g < (1 - \varepsilon_g) \Phi$.

Assuming $N$ fixed, the conditions for this cyclic behavior are as follows. If from collective fair play the greedy player achieves their demand (i.e., $1 \geq p_g^{(k-1)} = \theta_g^*/(1 - 1/N)^{N-1}$) then we require that the vigilante player receives significantly less than their fair share (i.e., $\theta_a^{(k-1)} = (1 - p_g^{(k-1)})(1 - 1/N)^{N-2}/N < (1 - \varepsilon_a) \Phi$), equivalently:

$$(1 - 1/N)^{N-1}[1 - (1 - \varepsilon_a)(1 - 1/N)] < \theta_a^*.$$  

Let $p_a^*, p_g^*$ be the interior Nash equilibrium of the game where the greedy and vigilante players maximize their respective net utilities, (1) and (2). We require the throughput of the greedy player

$$\Theta(p_g^*, p_a^*) < (1 - \varepsilon_a) \Phi.$$  

Suppose the vigilante utility $u_a(\theta) = U_a \log(1 + \theta)$ so that condition (4) becomes

$$-1 + U_a/M = \Phi.$$  

So, for a given $N$, if the set of three parameter

$$\theta_a^*, \varepsilon_a, \varepsilon_g$$

satisfy all three conditions (5), (6), and (7), then cyclic behavior will ensue.

III. NUMERICAL STUDY

A numerical study considering different cases was conducted. The player throughputs for some of the numerical results can be found in Table I, where $\theta_{N_i}, \theta_g$, and $\theta_a$ respectively are the average throughput for a normal, greedy, and vigilante player in the time interval of the study. If one greedy and one vigilante player are in play, note how the throughput for the normal players dramatically reduces as their numbers increase from Game A to Game B. For a total of $N = 8$ players, Games C involves two greedy and one
Fig. 1. $\epsilon_g=0.9, \epsilon_a=0.1$, $N=3$, $\theta_g^*=0.2$, $N = 3$

Fig. 2. $\epsilon_g=0.9, \epsilon_a=0.1$, $\theta_g^*=0.2$, $N = 3$

Fig. 3. $\epsilon_g=0.6, \epsilon_a=0.3$, $\theta_g^*=0.3$, $N = 6$

Fig. 4. $\epsilon_g=0.6, \epsilon_a=0.3$, $\theta_g^*=0.3$, $N = 6$

Fig. 5. $\epsilon_g=0.6, \epsilon_a=0.3$, $\theta_g^*=0.2$, $N = 8$

Fig. 6. $\epsilon_g=0.6, \epsilon_a=0.3$, $\theta_g^*=0.2$, $N = 8$
vigilante player, while Game D involves one greedy and two modified vigilante players with $T_a = 6$. For Game D, deadlock ensued without modified vigilante play.

The feasible parameter range of $(\epsilon_g, \epsilon_a, \theta^*_g)$ for cyclic behavior is depicted in Figure 7. Outside of the feasible range, the result is typically capture or near-capture of the channel by the greedy player(s).