Homework 4 Solution

1.(15pts.)

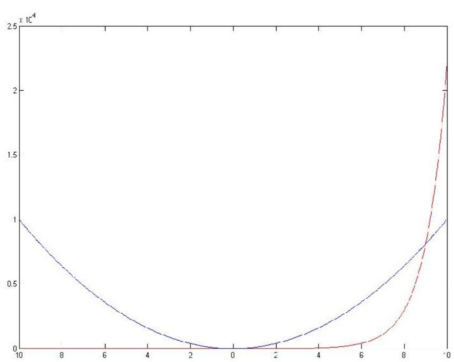
```
function root = Bisection(a,b)
tol=1/8;

while ((b-a)>tol)
    m=a+(b-a)/2;
    f_a=3*a^3-5*a^2-4*a+4;
    f_m=3*m^3-5*m^2-4*m+4;

    if (sign(f_a)==sign(f_m))
        a=m;
    else b=m;
    end
end
root=m;
fprintf('the root is found in the interval [ %g, %g]', a, b );

Output:
>> root=Bisection(0,1)
the root is found in the interval [ 0.625, 0.75]
root =

   0.62500000000000
```

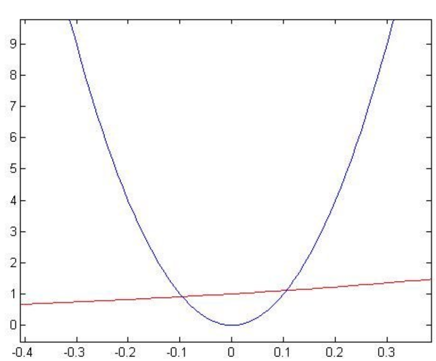
Achieving an error tolerance of 10^(-6) requires $\left\lceil log_2(\frac{b-a}{10^{-6}}) \right\rceil$=20 iterations.

2. (20pts.)

Firstly, we simply plot exp(x) Vs 100*x^2:

```
x=-10:.01:10;
y1=exp(x);
y2=100*(x.^2);
plot(x,y1,'r-',x,y2,'b-');
```





We can learn from the plot that probably there are 3 intersection points.

To verify this, we got f(-1) < 0, f(0) > 0, f(1) < 0, f(10)>0. 3 sign changes means there are at least 3 solutions. Plus the 3rd order derivative of exp(x)-100x^2 is positive whatever x is, so there are at mot 3 roots.

We implement the *Bisection.m* again for the following three intervals (-0.2, 0), (0, 0.2), (9, 10).

```
function root = Bisection(a,b)
tol=1e-3;

while ((b-a)>tol)
    m=a+(b-a)/2;
    f_a=exp(a)-100*(a^2);
    f_m=exp(m)-100*(m^2);

    if (sign(f_a)==sign(f_m))
        a=m;
    else b=m;
    end
end
root=m;
fprintf('the root is found in the interval [ %g, %g]', a, b );
```

```
>> Bisection(-0.2,0)
the root is found in the interval [ -0.0960938, -0.0953125]
ans =

   -0.0961

>> Bisection(0,0.2)
the root is found in the interval [ 0.104688, 0.105469]
ans =

    0.1055

>> Bisection(9,10)
the root is found in the interval [ 9.99902, 10]
ans =

    9.9990
```

3. (15pts.)

```
x=0.7;
for i=1:4
    x=x-((3*x^3-5*x^2-4*x+4)/(9*x^2-10*x-4));
end
```

1^{st} iteration: x = 0.6665
2^{nd} iteration: x = 0.6667
3^{rd} iteration: x = 0.6667
4^{th} iteration: x = 0.6667

4. (15pt.)

$1/x - c = 0$;
By Newton's method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{\frac{1}{x_n} - c}{-\frac{1}{x_n^{\ 2}}} = x_n(2 - cx_n).$$

Thus $x_{n+1} - \frac{1}{c} = x_n(2 - cx_n) - \frac{1}{c} = -c(x^2 - \frac{2x_n}{c} - \frac{1}{c^2}) = -c(x_n - \frac{1}{c})^2 < 0, \quad x_{n+1} < \frac{1}{c}.$

If $x_n < \frac{1}{c}, \quad x_{n+1} = x_n(2 - cx_n) > x_n(2 - c(1/c)) = x_n.$

Therefore, for n >= 1, x is an increasing sequence which converges to 1/c.

5. (15pt.)

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n) = x_n - \frac{x_n - x_{n-1}}{(\frac{1}{x_n} - c) - (\frac{1}{x_n - 1} - c)}(\frac{1}{x_n} - c) = x_n + x_{n-1} - cx_n x_{n-1}.$$

x(0) = 0.1;
x(1) = 0.2;
x(2) = 0.16;
x(3) = 0.136;
x(4) = 0.1437;
x(5) = 0.1429; fourth iteration

6. (20pt.)

Script:

```
x(1)=0.8;
y(1)=0.8;

for i=1:2
    f1=x(i)^4+x(i)*y(i)^2+y(i)^4-1;
    f2=x(i)^2+x(i)*y(i)-y(i)^2/4-1;

    f1x=4*x(i)^3+y(i)^2;
    f1y=2*x(i)*y(i)+4*y(i)^3;

    f2x=2*x(i)+y(i);
    f2y=x(i)-y(i)/2;

    h=(-f1*f2y+f2*f1y)/(f1x*f2y-f1y*f2x);
    k=(f1*f2x-f2*f1x)/(f1x*f2y-f1y*f2x);

    x(i+1)=x(i)+h;
    y(i+1)=y(i)+k;
end
```

Finally we get:

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0.7614 \\ 0.7317 \end{pmatrix}; \quad \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0.7622 \\ 0.7197 \end{pmatrix}.$$